FINITE ELEMENT APPROACH ON MHD FLOW THROUGH POROUS MEDIA PAST AN ACCELERATED VERTICAL PLATE IN A THERMALLY STRATIFIED FLUID

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ABSTRACT:
The current examination manages the investigation of unsteady MHD over permeable earlier media an enhanced vertical plate in a thermally stratified fluid. The principal equations are resolved by Galerkin finite-element scheme with help of Crank-Nicolson technique by running MATLAB Program. The impacts of the flow factors on velocity and temperature profiles are discussed with the help of graphs and table.

Key Words: MHD, Unsteady flow, FEM, Stratified fluid.

I. INTRODUCTION
Study of MHD flow with heat and mass transmission play a vital part in engineering & Sciences. It has many imperative applications, Ex. power generation system, cooling the nuclear reactors, liquid metal’s fluid, geothermal energy. Some of the investigators are studied on these like Basant Kumar [1] has contemplated MHD free convection & mass transform flow through permeable medium. Singh A. K [2] has considered transient natural convection between two vertical walls heated/cooled unevenly. MHD & Radiation effects on moving isothermal vertical plate by variable mass diffusion was studied by Muthucumaraswamy and Janakiraman [3]. Rajput and S. Kumar [4] have studied unsteady MHD normal convection flow earlier an enhanced vertical plate a thermally stratified fluid through porous media. Shankar Goud and Raja Shekar [6] have discussed finite element solution of viscous dissipative Variable temperature and mass diffusion influences unsteady MHD flow past a parabolic happening vertical plate. Jaiswal & Soundalgekar [6] investigated special effects on flow previous an infinite permeable oscillating plate through continuous pressure and embedded in permeable medium. Hence, this paper investigates unsteady MHD natural convection flow of stratified fluid through permeable media past an accelerated vertical plate. The nondimensional governing equations are resolved using the efficient finite process.

II. MATHEMATICAL FORMULATION
In this study, natural convective flow of gelatinous incompressible, electrical directing fluid is considered. The X-axis and the plate is taken along same direction i.e. upward and the plate is taken perpendicular to Y-axis. Fluid & plate are rest initially the time 0 ≤ t ≤ 0 , and the fixed temperature T′₀. At the time T′ > 0, Temp of plate increased or decreased T′₀ & the concentration level nearby the plate increasing linearly with time. That The following set of equations is the flow modal:

\[ \nu \frac{\partial^2 u'}{\partial y^2} - \frac{\partial u'}{\partial t'} + g \beta (T' - T'₀) - \left( \frac{\sigma B^2}{\rho} + \frac{\nu}{K'} \right) u' = 0 \quad \ldots (1) \]

\[ \frac{\partial T'}{\partial t'} = \alpha \frac{\partial^2 T'}{\partial y^2} - \gamma u' \quad \ldots (2) \]

Where \( \gamma = \frac{dT}{dz} + \frac{g}{C_p} \)
Here \( \frac{dT_\infty}{dz} \) is thermal stratification and \( \frac{g}{C_p} \) is pressure work term. 

The B.C.’s are:
\[
\begin{align*}
  t' &\leq 0 : u' = 0, T' = T_\infty' \\
  t' &> 0 : u' = A t', T' = T_W' , \quad \text{at } y' = 0 \quad \ldots (3) \\
  u' &\to 0, \quad T' \to T_\infty', \quad y' \to \infty
\end{align*}
\]

Where \( A(>0) \) is the constant acceleration, \( T_W' \) are the temperature & concentration of wall respectively, \( t' \) the time.

Introducing the following dimensionless quantities,
\[
y = \left( \frac{A}{v} \right)^2 y', \quad \theta = \frac{T' - T_\infty'}{T_W' - T_\infty'} \cdot \text{Pr} = \frac{\mu C_p}{k}, \quad K = \frac{K' A}{v}, \quad t = t'A,
\]

\[
S = \frac{\gamma}{A(T_W' - T_\infty')}, \quad \text{Gr} = \frac{\rho \beta (T_W' - T_\infty')}{A}, \quad M = \frac{\sigma B_0^2}{\rho A}, \quad \mu = \rho u
\]

With the help of dimensionless quantities, equations (1) and (2) becomes
\[
\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial t} - \left( \frac{M + 1}{K} \right) u + Gr \theta = 0 \quad \ldots (5) \quad \frac{\partial^2 \theta}{\partial y^2} - \text{Pr} \frac{\partial \theta}{\partial t} + S \text{Pr} u = 0 \quad \ldots (6)
\]

And nondimensional boundary conditions are
\[
\begin{align*}
  u &= 0, \quad \theta = 0 \quad \text{for all} \quad y \quad t \leq 0 \\
  u &\to t, \quad \theta = 1 \quad \text{at} \quad y = 0 \quad \ldots (7) \\
  u &\to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty \quad t > 0
\end{align*}
\]

III. METHOD OF SOLUTION

The determinate element strategy has been applied, get the mathematical results of equation (5) and (6) under margin circumstances (7). Given method is particularly effective & permits vigorous results of nonlinear differential systems. The following steps are:

Step 1: Discretization of the condition into components
Step 2: Derivation of the component conditions

The Derivation of constrained component conditions i.e., logarithmic conditions among the not known parameters of the limited component estimation, includes the accompanying 3 steps.

- Create the variational definition of the distinction condition.
- Suppose the inexact arrangement over distinctive finite component.
- Derive the determinate component conditions by replacing the estimated arrangement into the variational detailing.

Step 3: Get together of component conditions.
Step 4: Impositions of Boundary Conditions.
Step 5: Solution of the assembled equations.

By implementing the step (1-3) to the nonlinear differential equation (5), the assembled element conditions for 2 consecutive elements \( y_{i-1} < y < y_i \) & \( y_i < y < y_{i+1} \) put row consistent to the node ‘ \( i \) ’ to ‘ \( 0 \)’, with the variance schemes by taking \( h = l(e) \) the following obtained

\[
\frac{1}{6} \begin{bmatrix}
  u_i \ i-1 & 4 u_i \ i & u_i \ i+1 & + \frac{1}{h^2} \left[ -u_{i-1} + 2u_i + u_{i+1} + 4u_i + u_{i+1} + u_i \right]
\end{bmatrix} + \frac{N}{6} \begin{bmatrix}
  u_i \ i-1 & 4u_i \ i & u_i \ i+1 \end{bmatrix} = P \ldots (8)
\]

Implementing the trapezoidal law, the givenset of Equ’s in Crank-Nicholson method are achieved

\[
A_1 u_{i-1} + A_2 u_i + A_3 u_{i+1} = A_4 u_i \ i-1 + A_5 u_i + A_6 u_i \ i+1 + P* \ldots (9)
\]
\[ A_1 = A_3 = 2 - 6r - kN, \quad A_4 = A_6 = 2 + 6r - kN, \quad A_2 = 4(2 + 3r + kN), \]

\[ A_5 = 8 - 4kN - 12r, P^* = 12kP = 12kGr\theta_i^j, \quad N = \left( M + \frac{1}{K} \right) \]

Apply the same strategy, the following is obtained

\[ B_1\theta_i^{j+1} + B_2\theta_i^j + B_3\theta_i^{j+1} - P^{**} = B_4\theta_i^{j-1} + B_5\theta_i^j + B_6\theta_i^{j+1} \quad \text{... (10)} \]

where

\[ B_2 = 4Pr + 6r, \quad B_5 = 4Pr - 6r, P^{**} = -6kPr\theta_i^j \]

Here, \( r = k/h^2 \), time and \( y \) direction the mesh size are \( k \& h \) correspondingly. The given \((i, j)\) indicates space & time. In the Equ’s (9) – (10), taking \( i = 1, \ldots, n \) & utilizing boundary circumstances (7), the givenequ’sareattained:

\[ A_iX_i = B_i, \quad i = 1, \ldots, n \quad \text{... (11)} \]

Where the order matrix and the matrices of the columns have elements. Thomas algorithm method using for velocity, concentration & temp the results of the equations systems above are obtained. The mathematical solutions often attained by running this MATLAB software with reduced values of end. There was not a big improvement in \( u, \theta, \) then the Galerkin finite element method is constant & convergent.

Then the dimensionless skin friction is given by

\[ \tau = \left( \frac{\partial u}{\partial y} \right)_{y=0} \]

The dimensionless Nusselt number is given by

\[ \tau = \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \]

Fig. 1: velocity v/s magnetic field parameter Fig. 2: velocity v/s permeability parameter
Fig. 3: Velocity $v$ vs $t$

Fig. 4: Velocity $v$ vs stratification parameter

Fig. 5: Temperature $V$ vs magnetic field

Fig. 6: Temperature $v$ vs permeability parameter

Fig. 7: Temperature $v$ vs stratification parameter

Fig. 8: Temperature $v$ vs time
IV. RESULTS AND DISCUSSION
The velocity & temperature profiles are depicted for various flow factors like thermal Grashof number, stratification parameter, permeability parameter, magnetic field parameter and time in figures 1-8 and the skin friction and Nusselt number results are tabulated in Table-1. The impact magnetic field factor on the velocity & temperature profiles are shown in Fig. 1 & 5. Found the velocity diminishes & temperature increases with an increase of magnetic field limitation. Figures 2 & 6 explains the behaviour of the temperature & velocity for various values of permeability limitation. When increasing the values of permeability parameter the velocity increases and temperature drops. The effect of time on velocity & temperature summaries illustrated in Fig’s 3 & 8. It’s clear and the velocity profiles of the fluid rise and temperature fall down with an increase of time. Figures 4 and 7 displays the effects of the stratification parameter on velocity & temperature summaries. The velocity & temperature decrease with increasing the stratification parameter. The skin friction and Nusselt number are revealed in Table 1 for several values of flow parameters. Skin friction & Nusselt number increase when increasing values of the magnetic field factor & stratification parameter. But Skin friction and rate of heat transfer decrease with rise in thermal Grashof number and time.

V. CONCLUSION
The present study gives the following conclusions areas:
- Velocity raise with the increase in Gr, S, K and t and whereas decreases with increase M.
- With increase of M and S, temperature of the fluid reduces.
- Skin friction increases with an increase of M, S, K and t but decreases when thermal Grashof number is increased.
- With an increase of M, Gr K, Sand t the Nusselt number increases.

VI. REFERENCES: