

Review Article

COOPERATIVE ADVERTISING IN A TWO-ECHELON SUPPLY CHAIN BY CONSIDERING CAPACITY AND BUDGET CONSTRAINTS: A BI-LEVEL PROGRAMMING APPROACH

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Received: 10 Aug 2019 Revised and Accepted: 26 Oct 2019

ABSTRACT

Advertising is one of the ways that supply chains use to increase their customers. Cooperative advertising is one of the new approaches in advertising. In cooperative advertising, the manufacturer pays part of the advertising costs of its retailers. This increases retailers' incentives to invest more in advertising, which increases customer absorption. On the other hand, this advertisement is in the form of a contract that the manufacturer uses as a lever. Production capacities and available budgets of the manufacturer and retailers are some of the challenges of cooperative advertising in determining the optimal values of variables. Retailers compete with each other in the form of Generalized-Nash game to get more participation rates and more goods from the manufacturer. Because some variables are already predetermined by the manufacturer, the competition between the manufacturer and the retailers is a Stackelberg game. Finally, the mathematical model proposed and analyzed through numerical experiments, and some managerial insights are reported.

Keywords: Cooperative advertising; Generalized-Nash game; Stackelberg game; Variational inequalities

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DOI: <http://dx.doi.org/10.22159/jcr.07.01.01>

INTRODUCTION

One of the critical areas in today's business is Advertising (Yang, 2018). In general form, advertising can be divided into several areas. First, a company or organization seeking to increase sales of products has a specific focus on branding. In this study, these types of advertising are called global advertising. The second one, which is more recent and more popular than the former, is local advertising. Local advertising is also a type of advertising that is performed by a retailer or local distributor on a limited geographic scale. For example, car companies provide test drivings or food companies install advertising billboards at the showroom or allow customers to test the food products. An important issue that has been prevalent recently is cooperation in advertising. A manufacturer who is seeking to sell his/her product to a retailer proposes a participation rate under a partnership agreement that manufacturer commits itself to reimburse some of the costs of the retailer in local advertising, referred to as the manufacturer's participation in local advertising (Chutani and Sethi, 2018). Therefore, this will increase the retailer's willingness to invest more in local advertising, and consequently boost sales and profits for the manufacturer and the retailer more in comparison with non-cooperative conditions. Conversely, the retailer will also accept certain privileges for the manufacturer, such as considering a specific percentage of the exhibition for the manufacturer's product. Both of them make benefit of this contract and share the obtained profit with each other by some approaches (Zhao et al., 2019). However, what is the point of splitting the advertising into two categories of global or branding and local advertising, and why do we need both? Manufacturer's global advertising increases potential customers on a relatively large geographic scale whose motivation and willingness to purchase a promoted or branded product has increased, but it will not immediately. Local advertising complements global advertising and the intermediary link between intrinsic motivation and real shopping.

Potential customers who are influenced and motivated by global advertising, become real and final customers, influenced by local advertising and retailer recommendations and closer product experience. Therefore, every aspect of the advertising campaign, from the desire to the final stage to the purchase, is essential. It should be noted that, without local advertising, global advertising will have customers for us, as well as global advertising without local advertising. However, the result and high effectiveness we expect from these ads will not happen. So it is worthwhile to use both of these ads to complement each other (Amirtaheri et al., 2018). The supply chain considered in this study consists of a manufacturer that sells its product through a set of retailers. The retailers each have their monopoly market and have no involvement in the other retailer markets.

The manufacturer and each of the retailers have a certain amount of advertising budget available. From a production perspective, the manufacturer has a maximum production capacity that is related to the technical constraints of production. In these conditions, the goal of the retailers or provincial agencies is to make more sales and, subsequently, more profits. Since they each cooperate with the manufacturer of the advertising contract, it is natural for each to seek promotional subsidies and more goods from the manufacturer to sale, despite limited resources. Therefore, competition is flowing between the retailers in using these shared manufacturer resources. The answer to this competition at the retailer's level should search in the form of a Generalized Nash equilibrium. On the other hand, the manufacturer first announces its participation rate in the advertising costs of each retailer and the approved sales prices for offline and online channels. Then the retailers make decisions about the number of goods they should purchase and investment they make in local advertising by consideration of the manufacturer's decision. There is a Stackelberg game, with the leadership of the manufacturer and following by the retailers in which the leader decides based on the results of his/her

decision on the retailers and their decisions. The manufacturer is also affected by these retailers' decisions. If we consider the total competition, the sum of these two competitions is the Generalized Nash-Stackelberg game.

The characteristic features of this research are the coupling of retailers due to shared resources and introducing the concept of advertising prerequisites. In such a condition, it is not possible to find an equilibrium solution between retailers through the Nash equilibrium, and it is necessary to refer to the Generalized Nash format, which has much more technical complexity. Accordingly, the overall structure of the game in this paper is a Generalized Nash-Stackelberg game. From a mathematical modeling perspective, in this bi-level programming, the manufacturer is at the top level, and at the bottom is a set of retailers, which have their optimization problems. All second-level optimization problems are coupled due to typical technical constraints, and finding an optimal solution for all retailers has considerable technical complexity.

Finally, if we summarize the model's performance, it would be that the manufacturer, as the leader in the game will be announced the rate of participation in local advertising and determine the amount of money it will invest in global advertising, thereby retailers will decide about the quantity of goods to be purchased from the manufacturer and the investment in local advertising; all of these decisions will be made taking into account the capacity and budget constraints of the manufacturer and the retailers.

LITERATURE REVIEW

Based on previous studies (Aust & Buscher, 2014), five primary research areas for cooperative advertising can be outlined:

- **Vertical cooperative advertising:** This part of cooperative advertising is the most popular topic among researchers who have published many papers in this field. The subject in this area is, as the title implies, mostly related to the manufacturer as upstream supply chain and retailer as downstream supply chain relationships, where the manufacturer proposes a participation rate for retailer advertising costs.
- **Cooperative advertising in franchising:** It has the same solution as above, except that it uses advertising campaigns that take place under a contract between cost and profit-sharing between the manufacturer and the retailer (Bhattacharyya & Lafontaine, 1995; Hempelmann, 2006; Sigué & Chintagunta, 2009).
- **Horizontal cooperative advertising vs. global advertising:** Unlike the previous two, some companies work together on one level of a supply chain and compete with each other. Manufacturers advertise together for mutual products that can be in a group instead of advertising individually (Bass et al., 2005; Crespi & James, 2007).
- **Cooperative advertising vs. predatory advertising:** Cooperative advertising typically raises the total demand for all contributors. However, in predatory advertising, each member tends to increase its market share and demand with the reduction of the market demand of other competitors.
- **Joint advertising decisions:** The latest group of cooperative advertising research concerns joint advertising decisions, in which members cooperate to decide on advertising costs to maximize shared benefits, which may be at one level of the supply chain or outside (Buratto & Zaccour, 2009).

The most related to the present study can be seen as the first option or vertical cooperative advertising and decisions about the relationship between the manufacturer and the retailer. Here, we attempt to review the research work of this category of studies and finally provide a comparative table between research papers in this and present the results of the present research. Cooperative advertising models were first introduced by (Berger, 1972) as a game theory, which later focused on the competition factor in these games, including competition at the manufacturer level, between the retailer level, and

between manufacturers and retailers. More than half of the authors have limited their analysis to only determining the optimal decisions on member advertising, although some of them considered other factors like goods and services pricing, quality, remanufacturing, and inventory control besides.

In the field of vertical advertising, most studies have considered one manufacturer on the first level. However, studies such as (KARRAY & ZACCOUR, 2007) and (Kim & Staelin, 1999) considered the second manufacturer includes both Nash competition between manufacturers themselves and the Stackelberg competition between manufacturer and retailers. (Chutani & Sethi, 2018) proposed a model includes several manufacture and retailer with the Stackelberg game. In each level exist a Nash game for manufacturers and retailers, also. Manufacturers offer different promotional partnerships for each retailer, and retailers choose which of the manufactures to buy and amount of investment in local advertising. There are three types of games: 1. Nash game between manufacturers, 2. Nash game between retailers and 3. Stackelberg game between manufacturers and retailers.

One of the topics discussed in this paper is global advertising or manufacturer branding, in which (J. Chen, 2010) and (Karray & Zaccour, 2005, 2006) consider both manufacturer branding and retail store branding, which have their unique effects on demand function. Another essential factor in cooperative advertising is the approach of financial participation, including contribution in costs or discount offers for the retailer. Alternatively, as suggested by (De Giovanni, 2014), the manufacturer increases the retailer's incentive to invest more with a profit-sharing contract. Of course, both of these methods are similar, and the difference is the type and timeframe of payment. Static models are taking into account competition between two manufacturers that belong to the first and third integrated research areas, vertical and horizontal cooperative advertising. For example (Bergen & John, 1997) proposed a model with two manufacturers and two retailers; each product has two features or two dimensions: first is related to retailers, and second is to manufactures. So, there are four modes that the demand of each influenced by the cooperative advertising between them. They researched the optimum participation rate and types of advertising politics between manufacturer and retailer. (Kim & Staelin, 1999) suggested a Stackelberg game with two manufacturers as a leader and two retailers as followers, with co-op advertising being considered as a direct manufacturer payment quota in the model. They also used "pas through rate" in their paper, which indicated how much a retailer would spend on features to more sales again, from manufacturer payment. Features include such as allocating more space, shelves, and discounts for customers. In another study, (KARRAY & ZACCOUR, 2007) considered a Stackelberg model involving two manufacturers and two retailers. In this research, the effects of manufacturer and retailer branding together. Retailer branding called "Store Branding". They examined the impact of cooperative advertising programs on manufacturer and retailers' incomes. Concerning identical manufacturers and their retailers, they find if the brand substitution rate is high enough, cooperative advertising programs are an example of the famous "Prisoners' Dilemma" problem, and if not, cooperative advertising programs tend to increase retailers' profits. (Naimi Sadigh et al., 2012) considered a supply chain includes a manufacturer and a retailer, which, a manufacturer, produces several products. Demand for each product is a function of price, and the amount of advertising spend on it. This paper used two power scenarios, one is in the hands of the manufacturer, and the other is the retailer, which led to the Stackelberg-manufacturer game and the Stackelberg-retailer game, respectively. This research is intended to try to select a specific production interval for all manufactured products.

Decisions made by the manufacturer include choosing the timeframe of production of the wholesale price of each product, and the retailer decides on the retail price and costs and invests in advertising for each product, then. (Farshbaf-Geranmayeh et al., 2018a) believed WTP or willingness to pay has a relation with cooperative advertising issues and showed advertising caused increasing this parameter. The games

considered based on a balance of power, such as the power of the manufacturer and the retailer were identical, which brought about Nash equilibrium, and the power of the manufacturer was higher than the retailer that led to the Stackelberg game. Subsequently, in related research, (Farshbaf-Geranmayeh et al., 2018b) considered one manufacturer and several retailers as an oligopoly market. The market for these retailers, unlike our research, has an impact on each other and is not exclusive. The manufacturer in this research model has several products, and the demand function is influenced by price and advertising. From the results of this paper, we can point out the decrease in retail prices due to the competition between them, which is desirable to the end customers. (Xiao et al., 2019) proposed a supply chain with several retailers whose demands on each retailer were influenced by global advertising and local retailer advertising, although the retailers themselves cooperated in advertising. One of the innovations of this paper is to relate the advertising effectiveness to the environment of each retailer, which is considered by uncertainty. The demand function for each retailer represented as a statistical distribution with uncertainty. In order to avoid price overlap with demand function, wholesale and retail prices have been considered as exogenous. Paper used vertical hybrid multilevel cooperative advertising game. The manufacturer has a vertical partnership with its retailers, and the retailers organize a union at the bottom of a horizontal partnership with each other that decides on the total order of the manufacturer. The functions used tend to increase the manufacturer and total retailers profits. Surplus profits created by forming a union among the retailers contributed by the Shapley method. In this regard, numerous studies on the supply chain competitions have been reviewed, including (T.-H. Chen, 2015; Karray, 2015; Karray & Martín-Herrán, 2009; Liu et al., 2014), which focus on horizontal collaborative advertising. (Dan et al., 2012) examines a conventional two-channel supply chain in which the retailer provides services and can make decisions about service and pricing strategies. They conclude that the retailer should increase the quality of its channel service and consequently increase the price, and the manufacturer's pricing strategy will depend on customer loyalty.

On the other hand, some studies have incorporated these concepts into closed-loop supply chain discussions and expanded the scope of research by considering the collection of received goods, remanufacturing, cooperative advertising, and selling again. For example, (JiaPing Xie et al., 2017) consider a closed-loop supply chain with one manufacturer and one retailer. The manufacturer considered in this study sells its products in two ways: In the first case, the manufacturer sells his products to the retailer at a wholesale price and the retailer sells them to the end consumers called "Offline Channel" and the second is where the manufacturer directly sells its products to end-users without the intervention of a retailer using internet called "Online Channel". The demand function considered in the paper is influenced by both advertising and selling prices of both offline and online channels, but the global and local advertising is not considered. It is assumed that a certain percentage of customers tend to shop offline, and the remaining percentage tends to shop online, and this is parameterized in mathematical relationships. Also, the demand for each channel decreases with the increase in the price of that channel and decreases with the increase in the price of the other channel with different coefficients of impact. (Zheng et al., 2019) examined a closed-loop three-echelon supply chain consisting of one manufacturer, one distributor, and one retailer. The supply chain considered in this research is a closed-loop supply chain whereby reused goods are returned to the original manufacturer for recycling and reproduction. Both centralized and decentralized cases are examined: In a decentralized state, the manufacturer produces new products, collects used products, and is responsible for the recycling of these collected products. In the decentralized model, the manufacturer is the leader and is followed by the distributor and the retailer (Bi-level Programming).

According to the discussed studies, the present research considers a supply chain with one manufacturer and two retailers that will compete on the benefits of advertising budget and capacity of the manufacturer. Retailers have limited advertising budgets. For convenience and comparison of the present research with other contemporary research results following table is prepared.

Table 1. Comparison of similar researches

Constraint		Number of			Advertising			Demand Function	Structure		Time Frame		Research	
has	has not	manufactures	retailers	products	local	global	cooperative	advertising	price	decentralized	centralized	dynamic		static
	■	1	1	1	■	■	■		-	■			■	(Jinxing Xie & Ai, 2006)
	■	1	1	1	■	■	■			■			■	(Yue et al., 2006)
	■	1	1	1	■		■			■		■		(Esmaeili et al., 2009)

■		1	1	1	■	■	■		-	■			■	(Ahmadi-Javid & Hoseinpour, 2012)
	■	1	1	1	■	■	■			■			■	(Aust & Buscher, 2012)
■		1	1	<i>M</i>	■					■			■	(Naimi Sadigh et al., 2012)
	■	1	1	1	■	■	■		-	■			■	(Zhang et al., 2013)
	■	1	1	2	■	■	■			■			■	(Taleizadeh & Charmchi, 2015)
	■	1	1	1	■		■			■			■	(JiaPing Xie et al., 2017)
■		1	<i>n</i>	1	■					■			■	(Amirtaheeri et al., 2018)
	■	1	<i>n</i>	<i>j</i>	■	■	■			■			■	(Farshbaf-Geranmayeh et al., 2018b)
	■	<i>m</i>	<i>n</i>	<i>j</i>			■		-	■	■	■		(Chutani & Sethi, 2018)
	■	1	<i>n</i>	1	■	■	■		-	■	■		■	(Zheng et al., 2019)
■		1	2	1	■	■	■		-	■			■	Present Research

α Initial market demand without advertising
 p Wholesale price
 p^1 Retail price
 P_e Online price
 P_t Offline price
 K Advertising impact factor

A Global advertising investment
 α Local advertising investment
 n^1 Global advertising impact factor
 n^2 Local advertising impact factor
 v Price impact factor
 β, σ Prices sensitivity parameters

MATHEMATICAL MODELING

The supply chain, as described before, includes a manufacturer that sells its products through retailers/provincial agents or online. Retailers/provincial representatives have been coupled in terms of

shared resources, and each retailer has its monopoly market in which other retailers do not interfere. The following figure illustrates this description:

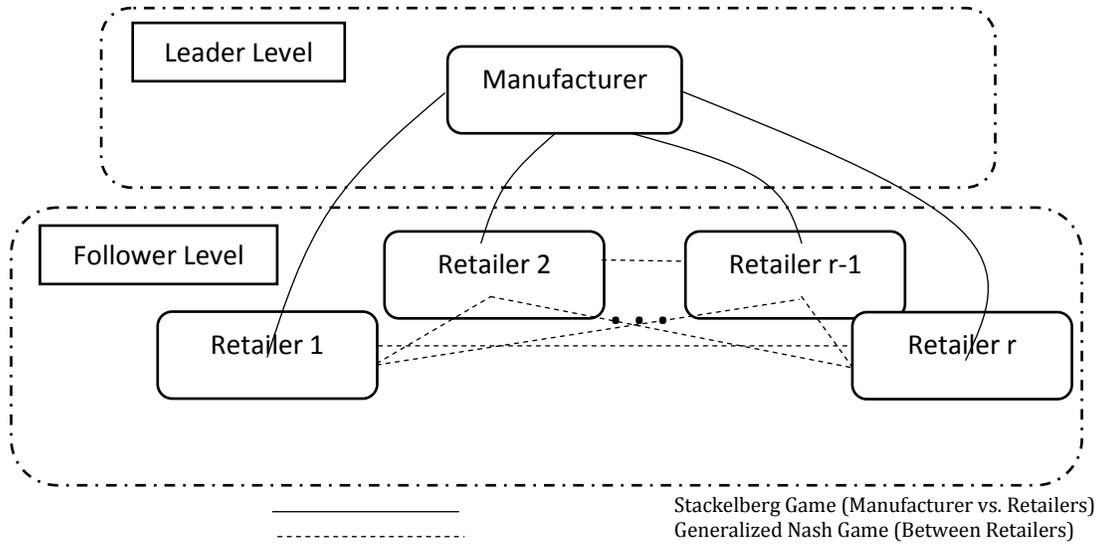


Figure 1. Games and relationships between supply chain's members

Sets

r Retailers

$$\sum_r X_r \leq \Omega \tag{2}$$

Parameters

- p_1 Margin profit for the manufacturer
- p_2 Margin profit for the retailer
- Ω Manufacturer maximum capacity
- ϕ The maximum budget allocated by the manufacturer for advertising
- γ_r Retailer r available budget for advertising
- Q_r The number of retailer r 's customers who typically buy the product from the retailer without advertising.
- n_1 The effect of global advertising on demand
- n_2 The effect of local advertising on demand

$$A + \sum_r \theta_r \alpha_r \leq \phi \tag{3}$$

$$X_r = Q_r + n_1 \sqrt{A} + n_2 \sqrt{\alpha_r} \tag{4}$$

$$X_r, A, \theta_r \geq 0 \tag{5}$$

$\forall r$

$$\begin{aligned} \text{Max } Z_r &= p_2 * X_r - (1 - \theta_r) \alpha_r \\ \text{s. t.:} & \\ (1 - \theta_r) \alpha_r &\leq \gamma_r \end{aligned} \tag{6}$$

$$\alpha_r \geq 0 \tag{8}$$

Leader (Manufacturer) under control variables

- X_r Shipped goods for the retailer r
- θ_r Manufacturer participation rate in the retailer r 's local advertising
- A Manufacturer investment in global advertising

Followers (Retailers) under control variables

- α_r Retailer r investment in local advertising

$$\text{Max } Z_M = \sum_r p_1 * X_r - \sum_r \theta_r \alpha_r - A \tag{1}$$

Equality 1 is the objective function, maximizes the profits of the manufacturer. Constraint 2 is the manufacturer's production capacity limitation that the total shipments to all retailers must not exceed this maximum capacity. Constraint 3 Apply to the manufacturer's allocated or available budget for the advertising sector, which includes the costs invested by the manufacturer in global advertising, and part of the local advertising costs of each retailer in which it participates. Constraint 4 shows that the demand function of retailer r , which is influenced by both local and global advertisings. $n_1 \sqrt{A}$ and $n_2 \sqrt{\alpha_r}$

are gained customers by global and local advertising, respectively. Equality 6 is the objective function of retailer r, which is equal to gained profits from sold goods minus the retailer's contribution to local advertising. The final inequality is constraint 8, which is guaranteed retailer investment in advertising will not exceed from his/her available budget.

SOLUTION METHODS

Bi-level optimization problems are commonly non-convex and non-differentiable and hard to solve intrinsically. In the best situation, we have linear phrases that have led to NP-hard problems (Colson et al., 2007). There are some methods to solve bi-level problems. If we have one follower and one leader, we can reformulate a bi-level problem to a single level problem using the Karush-Kuhn-Tucker (KKT) approach. In multi follower situation, we can use variational inequalities method with consideration of special necessary and sufficient conditions (like convexity and be differentiable in objective function and constraints) to transform the problem to a single level problem or using heuristic or metaheuristic algorithms to find suitable solutions (Gabriel et al., 2013). Absolutely not-linked followers yield simple Nash game and coupled followers create Generalized-Nash game with a solution space (it is possible to have several answers to the problem unlike simple Nash game).

Variational inequalities for retailers' level

Our objective function is minimization, x^* is the optimal solution when $f(x^*) \leq f(y)$ is true (In the maximization mode it is reverse). So we should have the following relationship:

$$\nabla f(x^*)^T (y - x^*) \geq 0 \tag{9}$$

Variational inequalities also aims to find the x^* that applies to the above relationship, and its general format is as follows:

$$G(x^*)^T (x - x^*) \geq 0 \tag{10}$$

As we can see from the above relationship, $G(x^*)^T$ is $\nabla f(x^*)^T$. This concept is genuine for finding Nash equilibrium, but in generalized Nash equilibrium, it is not always the same, and we may have different solutions and results. Our problem has a coupling constraint. If we formulate it as variational inequalities, we must first calculate $G(x^*)^T$. In the generalized Nash problems with the constrained player coupling constraints, we have two assumptions to consider:

- a. Consider the coupled constraint $\sum_r X^r \leq \Omega$ such that Ω is under the control of another agent. That agent provides resource Ω with an identical and fixed price for all retailers.
- b. Consider the coupled constraint $\sum_r X^r \leq \Omega$ such that player i believes that other players' decisions are outside the control of him/her. This constraint change to $X^r \leq \Omega - \sum_{i=r} X^i$ for player i which the right hand is as a parameter to him/her.

This research model is more in line with the assumptions of the first case, and the other agent or player is the same manufacturer. The price per unit consumption is the same as the wholesale price for all retailers. In this case, it can be demonstrated that the Variational Inequalities model for generalized Nash equilibrium will be as same as ordinary Nash equilibrium except that the coupled constraint will change the solution space. In the second case, we should also use the concept of Quasi-Variational inequalities.

To formulate problem with variational inequalities, we need two sets of L and G . The concept of G has already been explained, and L is the same solution space given the constraints of the problem:

$$G = \begin{pmatrix} \frac{\partial f_1}{\partial X_1} \\ \frac{\partial f_1}{\partial \alpha_1} \\ \vdots \\ \frac{\partial f_R}{\partial X_R} \\ \frac{\partial f_R}{\partial \alpha_R} \end{pmatrix} = \begin{pmatrix} p_2 \\ 1 - \theta_1 \\ \vdots \\ p_2 \\ 1 - \theta_R \end{pmatrix} \tag{11}$$

$$L = \left\{ X_r, \alpha_r \mid \begin{array}{l} \sum_r X_r \leq \Omega, \\ X_r = Q_r + m_2 \cdot n_1 \sqrt{A} + n_2 \sqrt{\alpha_r}, \\ (1 - \theta_r) \alpha_1 \leq \gamma_r \end{array} \right\} \quad \forall r \tag{12}$$

Now we write the Variational Inequalities relationships:

$$G(x^*)^T (x - x^*) = (p_2, 1 - \theta_1, \dots, p_2, 1 - \theta_R) \begin{pmatrix} X_1 - X_1^* \\ \alpha_1 - \alpha_1^* \\ \vdots \\ X_R - X_R^* \\ \alpha_R - \alpha_R^* \end{pmatrix} \geq 0 \tag{13}$$

Which becomes the following inequality:

$$\sum_{r=1}^R \{p_2 * (X_r - X_r^*) + (1 - \theta_r) * (\alpha_r - \alpha_r^*)\} \geq 0 \tag{14}$$

The inequality of number 14 creates the conditions, which we can have all optimal solution spaces for the retailer level, along with other related constraints. In the main bi-level problem, these inequalities are replaced with retailers' level by keeping the related constraints. Thus turning the bi-level problem into a single-level problem as follows:

(Note: variables X_r^* and α_r^* are the solutions of variational inequalities)

$$Max Z_M = \sum_r p_1 * X_r^* - \sum_r \theta_r \alpha_r^* - A \tag{15}$$

$$\sum_r X_r \leq \Omega \tag{16}$$

$$A + \sum_r \theta_r \alpha_r \leq \phi \tag{17}$$

$$X_r = Q_r + n_1 \sqrt{A} + n_2 \sqrt{\alpha_r} \quad \forall r \tag{18}$$

$$(1 - \theta_r) \alpha_r \leq \gamma_r \quad \forall r \quad (19)$$

$$\sum_{r=1}^R \{p_2 * (X_r - X_r^*) + (1 - \theta_r) * (\alpha_r - \alpha_r^*)\} \geq 0 \quad (20)$$

$$X_r, A, \alpha_r, \theta_r \geq 0 \quad \forall r \quad (21)$$

Table 2. Parameters values

Parameters	Value
p_1	10000
p_2	12000
Ω	15000
ϕ	15000000
γ_1	1500000
γ_2	1000000
Q_1	900
Q_2	400
n_1	1.6
n_2	1.5

Computational results and analysis

In this section, the effects of the parameters on the decision variables are discussed and analyzed. The model tested by numerical experiments includes one manufacturer and two retailers. Initial parameters considered for the model are presented in Table (2).

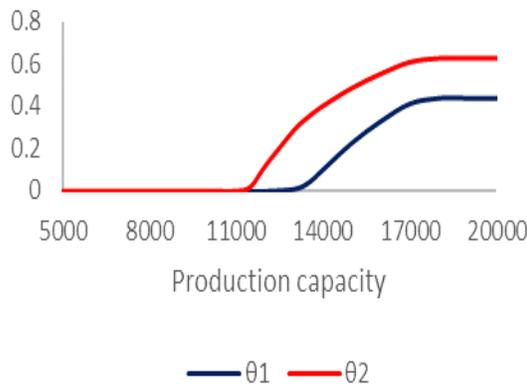


Figure 2. θ_1 and θ_2 changes according to the production capacity

As production capacity increases, participation rates start to increase and continue to grow as long as the budget constraint allows and attracting customers will be profitable (sales are higher than the cost of acquisition). Manufacturer at lower production capacities does not express partnership plans in local advertising, and in the higher production capacity, the manufacturer offers higher participation rates to use the capacity all. Partnership in local advertising starts where

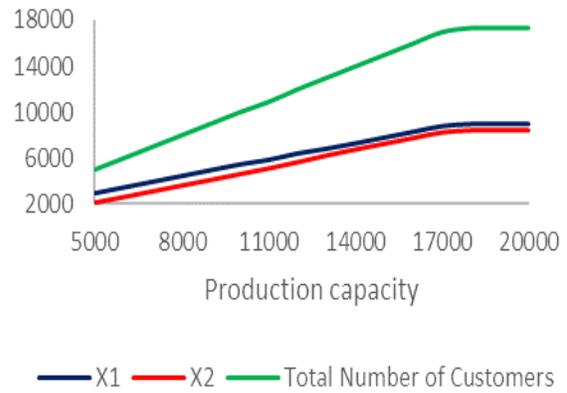


Figure 3. X_1, X_2 and the total number of customers changes according to the production capacity

retailers are not able to use their advertising capacity at their own cost. As the capacity has increased, efforts to attract customers have also increased. However, since the capacity of 18000 onwards, as the capacity has increased, absorbed customers have remained constant. This is due to the selling price of the product, which has far exceeded the cost of attracting new customers and will likely increase the manufacturer’s efforts to attract new customers as the price increases.



Figure 4. X_1 and X_2 changes according to the manufacturer’s available budget

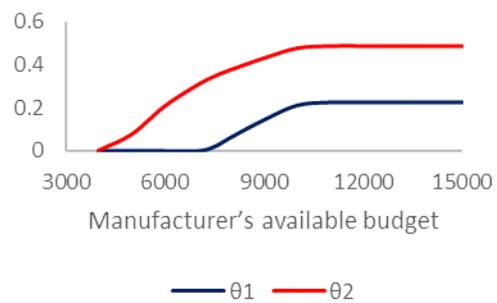


Figure 5. θ_1 and θ_2 changes according to the manufacturer’s available budget

Efforts to attract customers will increase as the manufacturer's available budget increases. These efforts continue until either the production capacity has reached its ultimate limitation or attracting the new customer is not profitable for the system. Naturally, the manufacturer's participation in local retailer advertising will also increase as the manufacturer's available budget increases. Why choosing the second retailer first, and then the first retailer for the

partnership goes back to the radical behavior of advertising influence. Investment initially has high impacts and jumps, and the slope of this impact is diminished by radical behavior. The second retailer has less available budget than the first retailer, and the manufacturer prefers to invest in second retailer local advertising in order to generate more profits. Next, as both retailers reach a level of incremental slope, the manufacturer will participate in close proportions to each other.

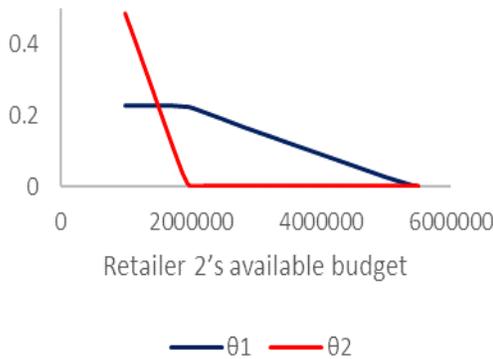


Figure 6. θ_1 and θ_2 changes according to retailer 2's available budget

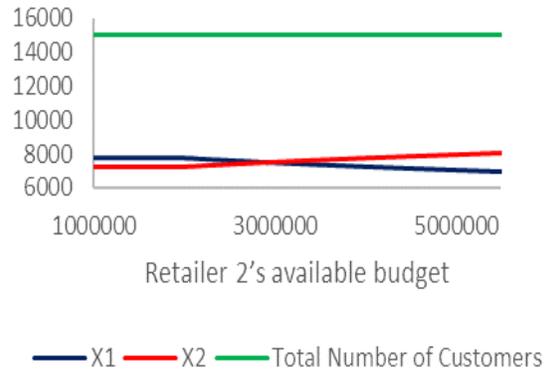


Figure 7. X_1 , X_2 and the total number of customers changes according to retailer 2's available budget

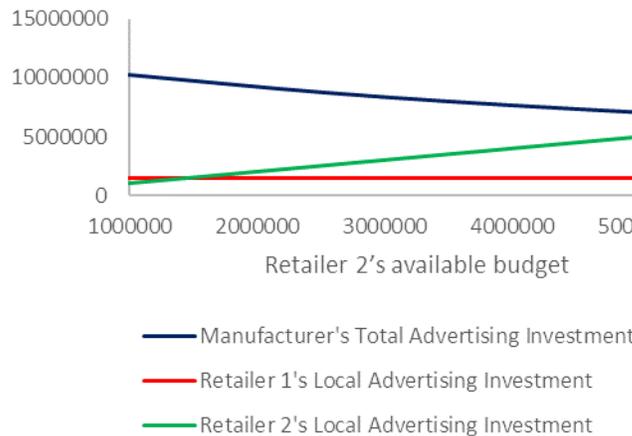


Figure 8. Manufacturer's total advertising investment and retailers' local advertising investment changes according to retailer 2's available budget

By increasing in second retailer budget, the manufacturer reduces its participation in the second retailer's local advertising. The manufacturer also reduces its participation in the first retailer's local advertising, but much less than the reduction in local first retailer's advertising. The second retailer has been able to reach more customers than the first retailer by increasing its budget. Because the production capacity is constant, the manufacturer has also reduced the amount of participation in the first retailer's local advertising by considering remained production capacity. Increasing the second retailer's available budget has not consistently increased the number of customers and used all of the budgets to attract customers, for two main reasons: 1-production capacity is constant and 2-attracting new customers not being profitable.

RESULTS

Increasing the manufacturer's available capacity or budget always had a positive effect on the total supply chain members. The point is that

each of these increases has reached a specific level and increased the revenue of members. In other words, the system will continue to do this so as long as it is profitable to attract new customers and may not utilize some of its capabilities. These optimal limits can be used in investment discussions or in determining the optimal production capacity. On the other hand, the more a retailer's budget or investment increases because the retailer has no other source of income, the manufacturer is confident that he/she will make every effort to increase demand in his/her area. As a result, as a retailer's budget increases, the manufacturer's participation in local cooperative advertising decreases. However, his/her earnings go up a bit, and after that, because of limited capacity, he/she does not spend any more on advertising. One of the strengths of a retailer that is evident in the charts is the number of customers who are willing to buy that product without advertising. This parameter is directly related to the population density of the area. So retailers in populated areas have more power and flexibility than others.

Future study

The results mentioned were related to retailers whose only source of income was from selling the manufacturer's product. In a way, it can be said that these retailers acted as sales agents in a particular geographic area. Next, it is necessary to involve other manufacturers' products in retail and to examine the results of that model. The model can also be examined at different time intervals where the model design needs to be dynamic, and the decisions of the manufacturer and retailers analyze at different times. Finally, some parameters in the model, such as capacity, price, and others, can be considered as variables so that the optimal values of these variables can be calculated and analyzed according to the model conditions.

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