

Bianchi Type VI_0 Bulk Viscous Fluid String Dust Magnetized Cosmological Model In S-B Theory

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Abstract: Bianchi type VI_0 magnetized bulk viscous fluid string dust cosmological model is investigated. To obtain a determinate model, we have assumed the conditions $\frac{\sigma}{\theta} = \text{constant}$ and $\zeta \theta = \text{constant}$, where σ is the shear, θ is the expansion of model and ζ the coefficient of bulk viscosity. The behavior of the model in the presence and in absence of magnetic field together with physical and geometrical aspects of the model are also discussed.

Keywords: Bianchi type model VI_0 , Bulk viscous, magnetized String model, S-B theory of gravity.

1 Introduction:

Recent measurements from some type Ia supernovae (SNe) at the intermediate and high red shift [1,2] indicate that the bulk of the energy in the universe is repulsive and appears like quintessence component that is an unknown form of dark energy (in addition to the ordinary CDM matter) probably of primordial origin [3]. Together with the observations of CBM anisotropies [4], such results seems to provided an important piece of information connecting an early inflationary stage with the astronomical observations. This state of affairs has stimulated the interest in more models containing an extra component describing the dark energy and simultaneously accounting for the present accelerated stage of universe.

In the early stage of universe when neutrino decoupling occurred, the matter behaved like a viscous fluid. The coefficient of viscosity decreases as universe expands. Padmanabhan and Chitre [5] have investigated the effect of bulk viscosity on the evolution of the universe at large stage. Verma and shri ram [6,7] have derived a bulk viscous Bianchi type III model of universe.

The field equations of Saez-Ballester [8] scalar tensor theory for the combined scalar and tensor fields are

$$G_{ij} - \omega \phi^n \left(\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -8\pi T_{ij} \quad (1.1)$$

and the scalar field ϕ satisfies the equation

$$2\phi^n \phi_{,i}^i + n\phi^{n-1} \phi_{,k} \phi^{,k} = 0 \quad (1.2)$$

Where $G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R$ is the Einstein tensor, R the scalar curvature, ω and n are constants, T_{ij} is the stress tensor of matter and comma and semicolon denote partial and covariant differentiation respectively.

Also we have energy conservation equation,

$$T_{;i}^{ij} = 0 \quad (1.3)$$

which is the consequences of the field equation (1) and (2).

The effect of viscosity on the evolution of the universe and the strong dissipation, due to the neutrino viscosity, may considerably reduce the anisotropy of black body radiation, which has been discussed by Misner [9,10]. Barrow [11], Padmanabhan and Chitre [12], Pavon et al. [13], Lima et al. [14], and Martens [15] are some of the authors who have studied the possibility of bulk viscosity leading to inflationary-like solution in general relativistic FRW models. The effective total negative pressure, which leads to a repulsive gravity in bulk viscosity, overcomes the attractive gravity of the matter and gives an impulsion for rapid expansion of the universe. Many authors like Bali and Pradhan [16], Tripathy et al. [17], Singh and Kale [18], Rao and Sireesha [19], and Shri Ram et al. [20] have explored cosmological models with bulk viscosity in different theories of gravitation. Rao et al. [21, 22] studied FRW bulk viscous cosmological model in some of scalar tensor theories of gravitation.

Tripathy et al. [17] studied string cloud cosmologies for Bianchi type-III models with electromagnetic field. Adhav et al. [23] discussed Bianchi type-III string cosmological models in general theory of relativity. Rao et al. [24] obtained

anisotropic universe with cosmic strings and bulk viscosity in a scalar–tensor theory of gravitation. Sagar et al. [25] explored Bianchi type-III bulk viscous cosmic string models in Brans–Dicke theory of gravitation.

Motivated by above work, we investigated the Bianchi type VI_0 model fluid in presence of magnetic field and bulk viscosity. In this paper, we present metric and field equations in section 2. We deal with solution of field equation in section 3. we also investigated physical and geometrical features of model in section 4. we also investigated some special cases in section 5 and concluding remark is given in section 6.

2 Metric and field equations :

We consider the spatial homogenous and anisotropic space-time represented by the Bianchi type VI_0 metric as

$$ds^2 = -dt^2 + a_1^2 dx^2 + a_2^2 e^{-2x} dy^2 + a_3^2 e^{2x} dz^2 \tag{2.1}$$

Where a_1, a_2, a_3 are functions of cosmic time t .

We consider the energy momentum tensor for strings dust is given by

$$T_i^j = \rho u_i u^j - \lambda v_i v^j - \zeta \theta (g_i^j + u_i u^j) + E_i^j \tag{2.2}$$

Where u_i and v_i satisfy the conditions

$$u_i u^i = -v_i v^i = -1 \text{ and } u^j v_j = 0 \tag{2.3}$$

Where $\rho = \rho_p + \lambda$ is rest energy density for a cloud of strings. Here ρ_p is the particle density, λ is the string tension density, u^i the flow velocity vector, v^i the direction of strings and ζ the coefficient of bulk viscosity. Here

E_{ij} is the electromagnetic field given by Lichnerowics

$$E_i^j = \bar{\mu} \left[|h|^2 \left(u_i u^j + \frac{1}{2} g_i^j \right) - h_i h^j \right] \tag{2.4}$$

Where u^i the flow velocity vector satisfying

$$g_{ij} u^i u^j = -1 \tag{2.5}$$

$\bar{\mu}$ is the magnetic permeability and h_i the magnetic flux vector defined by

$$h_i = \frac{\sqrt{-g}}{2\mu} \epsilon_{ijkl} F^{kl} u^j \tag{2.6}$$

Where F^{kl} is the electromagnetic field tensor and ϵ_{ijkl} is the Levi Civita tensor density. We assume the coordinates to be commoving so that

$$u^i = (0,0,0,1) \quad \text{and} \quad v^i = \left(\frac{1}{A}, 0, 0, 0 \right) \tag{2.7}$$

The incident magnetic field is taken along x-axis so that

$$h_1 \neq 0, \quad h_2 = 0 = h_3 = h_4 \tag{2.8}$$

The first set of Maxwell's equation

$$F_{ij;k} + F_{JK;l} + F_{kl;j} = 0 \tag{2.9}$$

Leads to

$$F_{23} = \text{constant} = H \text{ (say)} \tag{2.10}$$

Due to the assumption of infinite electrical conductivity of fields, we have

$$F_{14} = 0 = F_{24} = F_{34} \tag{2.11}$$

The only non-vanishing component of F_{ij} is F_{23} . Hence

$$h_1 = \frac{a_1 k}{a_2 a_3 \mu} \tag{2.12}$$

And

$$|h|^2 = \frac{k^2}{a_2^2 a_3^2 \mu^2} \tag{2.13}$$

From equation (2.4)

$$E_1^1 = \frac{-a_1^2 k^2}{2a_2^2 a_3^2 \mu} = -E_2^2 = -E_3^3 = -E_4^4 \tag{2.14}$$

Equation (2.2) leads to

$$T_1^1 = -\lambda - \zeta\theta - \frac{k^2}{2a_2^2 a_3^2} \tag{2.15}$$

$$T_2^2 = -\zeta\theta - \frac{k^2}{2a_2^2 a_3^2} \tag{2.16}$$

$$T_3^3 = -\zeta\theta - \frac{k^2}{2a_2^2 a_3^2} \tag{2.17}$$

$$T_4^4 = -\rho - \frac{k^2}{2a_2^2 a_3^2} \tag{2.18}$$

In co-moving coordinates system, Einstein field equation (1.1), for Bianchi type VI_0 metric (2.1) and the energy momentum tensor (2.2) leads to following system of equations

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{1}{a_1^2} - \frac{\omega}{2} \phi^n \dot{\phi}^2 = 8\pi \left[\lambda + \xi + \frac{k^2}{2a_2^2 a_3^2} \right] \tag{2.19}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{1}{a_1^2} - \frac{\omega}{2} \phi^n \dot{\phi}^2 = 8\pi \left[\xi - \frac{k^2}{2a_2^2 a_3^2} \right] \tag{2.20}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} - \frac{1}{a_1^2} - \frac{\omega}{2} \phi^n \dot{\phi}^2 = 8\pi \left[\xi - \frac{k^2}{2a_2^2 a_3^2} \right] \tag{2.21}$$

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{1}{a_1^2} + \frac{\omega}{2} \phi^n \dot{\phi}^2 = 8\pi \left[\rho + \frac{k^2}{2a_2^2 a_3^2} \right] \tag{2.22}$$

$$\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} = 0 \tag{2.23}$$

$$\ddot{\phi} + \dot{\phi} \left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right) + \frac{n}{2} \frac{\dot{\phi}^2}{\phi} = 0 \tag{2.24}$$

Where $\xi = \zeta\theta$

3.Solution of Field equation

From equation (2.23)

$$a_3 = \alpha a_2 \tag{3.1}$$

Where α is constant.

For model (2.1), we have

$$\theta = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \tag{3.2}$$

From equation (3.1) and (3.2), we get

$$\theta = \frac{\dot{a}_1}{a_1} + \frac{2\dot{a}_2}{a_2} \tag{3.3}$$

For a given metric (2.1), we have

$$\sigma = \sqrt{\frac{1}{3} \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} \right)} \tag{3.4}$$

To get deterministic model, we impose the condition

$$\frac{\sigma}{\theta} = \beta (\text{const } t) \tag{3.5}$$

Where β is constant, σ is shear and θ the expansion in the model

From the equation (3.3), (3.4) and (3.5)

$$\sqrt{\frac{1}{3}} \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2} \right) = \beta \left(\frac{\dot{a}_1}{a_1} + \frac{2\dot{a}_2}{a_2} \right) \tag{3.6}$$

Which yields to

$$a_1 = \gamma \tag{3.7}$$

$$a_2^2 = \rho \sin nh \left(2\sqrt{\frac{1}{\gamma^2} + m} \right) t \tag{3.8}$$

$$a_3^2 = \alpha^2 \rho \sin nh \left(2\sqrt{\frac{1}{\gamma^2} + m} \right) t \tag{3.9}$$

Where γ is constant and $m = 8\pi\xi$

Therefore, the metric (2.1) in the presence of magnetic field, bulk, viscosity, reduces to form

$$ds^2 = -dt^2 + \gamma^2 dx^2 + \rho \sin nh \left(2\sqrt{\frac{1}{\gamma^2} + m} \right) t e^{-2x} dy^2 + \alpha^2 \rho \sin nh \left(2\sqrt{\frac{1}{\gamma^2} + m} \right) t e^{2x} dz^2 \tag{3.10}$$

Which again leads to

$$ds^2 = -dt^2 + \gamma^2 dx^2 + \left[\frac{\left(\frac{4\pi k^2}{\alpha^2} - \frac{\omega c_2^2}{2\alpha^2 \gamma^2} \right)^{\frac{1}{2}}}{\frac{1}{\gamma^2} + 8\pi\xi} \right] \left\{ \sin nh \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t e^{-2x} dy^2 + \alpha^2 \sin nh \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t e^{2x} dz^2 \right\} \tag{3.11}$$

In the absence of magnetic field i.e. when $k \rightarrow 0$ then the metric (3.11) leads to

$$ds^2 = -dt^2 + \gamma^2 dx^2 + \left[\frac{-\frac{\omega c_2^2}{2\alpha^2 \gamma^2}}{\frac{1}{\gamma^2} + 8\pi\xi} \right]^{\frac{1}{2}} \left\{ \sin nh \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t e^{-2x} dy^2 + \alpha^2 \sin nh \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t e^{2x} dz^2 \right\} \tag{3.12}$$

In the absence of magnetic field i.e. when $\xi \rightarrow 0$ then the metric (3.11) leads to

$$ds^2 = -dt^2 + \gamma^2 dx^2 + \gamma \left[\left(\frac{4\pi k^2}{\alpha^2} - \frac{\omega c_2^2}{2\alpha^2 \gamma^2} \right)^{\frac{1}{2}} \right] \left\{ \sin nh \left(\frac{2}{\gamma} t \right) e^{-2x} dy^2 + \alpha^2 \sin nh \left(\frac{2}{\gamma} t \right) e^{2x} dz^2 \right\} \tag{3.13}$$

In the absence of magnetic field and bulk viscosity i.e. $k \rightarrow 0$ and $\xi \rightarrow 0$ then the metric (3.11) leads to

$$ds^2 = -dt^2 + \gamma^2 dx^2 + \left[\frac{-\omega c_2^2}{2\alpha^2 \gamma^2} \right]^{\frac{1}{2}} \left\{ \sin nh \left(\frac{2}{\gamma} t \right) e^{-2x} dy^2 + \alpha^2 \sin nh \left(\frac{2}{\gamma} t \right) e^{2x} dz^2 \right\} \quad (3.14)$$

4. Physical and Geometrical Features:

The energy density (ρ), the string tension density (λ), the particle density (ρ_p), the expansion (θ), the shear (σ), special volume (R^3), Hubble parameter (H) the deceleration parameter (q) for model (3.11) in presence of magnetic field and bulk viscosity are given by

$$8\pi\rho = \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \coth^2 \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) - \frac{1}{\gamma^2} + \frac{\omega c_2^2 \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \coth^2 \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) - \frac{4\pi k^2}{2\alpha^2 \gamma^2 \rho \sinh \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) - \alpha^2 \rho^2 \sinh^2 \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right)} \quad (4.1)$$

$$8\pi\xi = \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \left[2 - \coth^2 \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) \right] - \frac{1}{\gamma^2} - \frac{\omega c_2^2 \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \coth^2 \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) - \frac{4\pi k^2}{2\alpha^2 \gamma^2 \rho \sinh \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) - \alpha^2 \rho^2 \sinh^2 \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right)} \quad (4.2)$$

$$8\pi\lambda = 2 \left(\frac{1}{\gamma^2} + 8\pi\xi \right) + \frac{2}{\gamma^2} - \frac{8\pi k^2}{\alpha^2 \rho^2 \sinh^2 \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right)} \quad (4.3)$$

$$R^3 = 2 \alpha \gamma \rho \sinh \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) \quad (4.4)$$

$$\theta = 2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \coth \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) \quad (4.5)$$

$$H = \frac{2}{3} \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \coth \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) \quad (4.6)$$

$$q = 3 \operatorname{sech}^2 \left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi} t \right) - 1 \quad (4.7)$$

$$A_m = \frac{1}{2} \quad (4.8)$$

$$\sigma^2 = \frac{1}{3} \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \coth^2 \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t \tag{4.9}$$

5. Some special Cases:

Case I: In the absence of magnetic field i.e. when $k \rightarrow 0$

The energy density (ρ), the string tension density (λ), the particle density (ρ_p), the expansion (θ), the shear (σ), special volume (R^3), Hubble parameter (H) the deceleration parameter (q) for model (3.12) are given by

$$8\pi\rho = \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \coth^2 \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t - \frac{1}{\gamma^2} + \frac{\omega c_2^2 \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \coth^2 \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t}{2\alpha^2 \gamma^2 \rho \sinh \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t} \tag{5.1}$$

$$8\pi\xi = \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \left[2 - \coth^2 \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t \right] - \frac{1}{\gamma^2} - \frac{\omega c_2^2 \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \coth^2 \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t}{2\alpha^2 \gamma^2 \rho \sinh \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t} \tag{5.2}$$

$$8\pi\lambda = 2 \left(\frac{2}{\gamma^2} + 8\pi\xi \right) \tag{5.3}$$

$$R^3 = 2 \alpha \gamma \rho \sinh \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t \tag{5.4}$$

$$\theta = 2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \coth \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t \tag{5.5}$$

$$H = \frac{2}{3} \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \coth \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t \tag{5.6}$$

$$q = 3 \sec h^2 \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t - 1 \tag{5.7}$$

$$A_m = \frac{1}{2} \tag{5.8}$$

$$\sigma^2 = \frac{1}{3} \left(\frac{1}{\gamma^2} + 8\pi\xi \right) \coth^2 \left(2 \sqrt{\frac{1}{\gamma^2} + 8\pi\xi} \right) t \tag{5.9}$$

Case II: In the absence of bulk viscosity i.e. when $\xi \rightarrow 0$

The energy density (ρ), the string tension density (λ), the particle density (ρ_p), the expansion (θ), the shear (σ), special volume (R^3), Hubble parameter (H) the deceleration parameter (q) for model (3.13) are given by

$$8\pi\rho = \left(\frac{1}{\gamma^2}\right)\left(\coth^2\left(\frac{2}{\gamma}\right)t - 1\right) + \frac{\omega c_2^2\left(\frac{1}{\gamma^2}\right)\coth^2\left(\frac{2}{\gamma}\right)t}{2\alpha^2\gamma^2\rho\sinh\left(\frac{2}{\gamma}\right)t} - \frac{4\pi k^2}{\alpha^2\rho^2\sinh^2\left(\frac{2}{\gamma}\right)t} \quad (5.10)$$

$$8\pi\lambda = \frac{4}{\gamma^2} - \frac{8\pi k^2}{\alpha^2\rho^2\sinh^2\left(\frac{2}{\gamma}\right)t} \quad (5.11)$$

$$R^3 = 2\alpha\gamma\rho\sinh\left(\frac{2}{\gamma}\right)t \quad (5.12)$$

$$\theta = \frac{2}{\gamma}\coth\left(\frac{2}{\gamma}\right)t \quad (5.13)$$

$$H = \frac{2}{3\gamma}\coth\left(\frac{2}{\gamma}\right)t \quad (5.14)$$

$$q = 3\sec h^2\left(\frac{2}{\gamma}\right)t - 1 \quad (5.15)$$

$$A_m = \frac{1}{2} \quad (5.16)$$

$$\sigma^2 = \frac{1}{3\gamma}\coth^2\left(\frac{2}{\gamma}\right)t \quad (5.17)$$

Case III: In the absence of magnetic field and bulk viscosity i.e. when $k \rightarrow 0$ and $\xi \rightarrow 0$.

The energy density (ρ), the string tension density (λ), the particle density (ρ_p), the expansion (θ), the shear (σ), special volume (R^3), Hubble parameter (H) the deceleration parameter (q) for model (3.14) are given by

$$8\pi\rho = \left(\frac{1}{\gamma^2}\right)\left(\coth^2\left(\frac{2}{\gamma}\right)t - 1\right) + \frac{\omega c_2^2\left(\frac{1}{\gamma^2}\right)\coth^2\left(\frac{2}{\gamma}\right)t}{2\alpha^2\gamma^2\rho\sinh\left(\frac{2}{\gamma}\right)t} \quad (5.18)$$

$$8\pi\lambda = \frac{4}{\gamma^2} \quad (5.19)$$

$$R^3 = 2\alpha\gamma\rho\sinh\left(\frac{2}{\gamma}\right)t \quad (5.20)$$

$$\theta = \frac{2}{\gamma}\coth\left(\frac{2}{\gamma}\right)t \quad (5.21)$$

$$H = \frac{2}{3\gamma}\coth\left(\frac{2}{\gamma}\right)t \quad (5.22)$$

$$q = 3\sec h^2\left(\frac{2}{\gamma}\right)t - 1 \quad (5.23)$$

$$A_m = \frac{1}{2} \tag{5.24}$$

$$\sigma^2 = \frac{1}{3\gamma} \coth^2\left(\frac{2}{\gamma}t\right) \tag{5.25}$$

6. Conclusion:

We discuss the Bianchi type VI_0 model in presence of magnetic field and Bulk viscosity. In the presence of magnetic field and Bulk viscosity, the model (3.11) starts with a big bang. The expansion in the model decreases as the time increases. The special volume increases as time increases. When $3 \sec h^2\left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi}\right)t < 1$, then $q < 0$ and

when $3 \sec h^2\left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi}\right)t > 1$, then $q > 0$ hence model (3.11) represent an accelerating and decelerating universe respectively. The energy density $\rho \rightarrow \infty$ when $t \rightarrow 0$.

In the absence of magnetic field, the model (3.12) starts with a big bang. The expansion in the model decreases as the time increases. The special volume increases as time increases. When $3 \sec h^2\left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi}\right)t < 1$, then $q < 0$ and

when $3 \sec h^2\left(2\sqrt{\frac{1}{\gamma^2} + 8\pi\xi}\right)t > 1$, then $q > 0$ hence model (3.12) represent an accelerating and decelerating universe respectively. The energy density $\rho \rightarrow \infty$ when $t \rightarrow 0$.

In the absence of bulk viscosity, the model (3.13) starts with a big bang. The expansion in the model decreases as the time increases. The special volume increases as time increases. When $3 \sec h^2\left(\frac{2}{\gamma}\right)t < 1$, then $q < 0$ and when

$3 \sec h^2\left(\frac{2}{\gamma}\right)t > 1$, then $q > 0$ hence model (3.13) represent an accelerating and decelerating universe respectively.

The energy density $\rho \rightarrow \infty$ when $t \rightarrow 0$.

In the absence of magnetic field and bulk viscosity, the model (3.14) starts with a big bang. The expansion in the model decreases as the time increases. The special volume increases as time increases. When $3 \sec h^2\left(\frac{2}{\gamma}\right)t < 1$, then $q < 0$

and when $3 \sec h^2\left(\frac{2}{\gamma}\right)t > 1$, then $q > 0$ hence model (3.14) represent an accelerating and decelerating universe

respectively. The energy density $\rho \rightarrow \infty$ when $t \rightarrow 0$. $\frac{\sigma}{\theta}$ remains constant throughout.

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