Neutrosophic Mixed Graph and its Application in Economic Competition Among Countries

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Abstract

Competition is running everytime everywhere. But economic competition is one of the important part for the countries. In this study, we represent competitions using graph theory. To do this we introduce neutrosophic mixed graph and related terms. The competitions on neutrosophic mixed graphs are represented with algorithms. Also, the m −step neutrosophic graphs and competitions are studied. At last, we presented the economic competition among various countries including India. To represent this, we considered G.D.P. (Gross Domestic Product), G.P.I. (Global Peace Index) and D.R.I. (Disaster Risk Index) of all countries and found the result.

Keywords: Mixed graphs, neutrosophic graphs, neutrosophic mixed graphs, competition graphs, m −step competition graph.

1. Introduction

Graph theory is a crucial part of the demonstration of various networks [3]. So it is applied to model a bond between a specific set of items. A node characterizes each object, and an edge characterizes their relationship. The great mathematician Euler initially provided the notions of graphs in 1736 by resolving the famous Konigsberg bridge problem. There are numerous applications [1] of graph theory in science and engineering. Therefore several developments of a graph have been established over the years. The competition graph is an important branch of graph theory. The study of competition was started with the application in the food web of the ecosystem by Cohen (1968). Then there were various studies on crisp competition graphs [3,4], competition numbers [5,6] and step competition graph [7,8].

In real life, having such a situation of imprecise data [9] and using these concepts, Kaufman (1973) defined fuzzy graph [10] where the degree of memberships of all vertices and edges considered. The notions of the fuzzy conditions in the food web investigated by Samanta et al. in 2013 and designed fuzzy competition graph [11] more appropriately. Samanta et al. in 2016 and 2018 recommended the generalized fuzzy graph [12] and generalized fuzzy competition graph [13]. Mahapatra and Samanta [26-34] presented many applications of the fuzzy graph.

Smarandache (1998) proposed the generalization from concepts of fuzziness to neutrosophic and introduced neutrosophic set [14]. Broumi et al. in 2015 presented the single-valued neutrosophic graph [15]. A graph $G = (V, E)$ where $E \subseteq V \times V$ is said to be a neutrosophic graph if

i) there exist functions $\alpha_T: V \to [0,1], \alpha_F: V \to [0,1]$ and $\alpha_I: V \to [0,1]$ such that

$$0 \leq \alpha_T(v_i) + \alpha_F(v_i) + \alpha(v_i) \leq 3 \text{ for all } v_i \in V \ (i = 1,2,3, \dots, n)$$

where $\alpha_T(v_i), \alpha_F(v_i), \ \alpha_I(v_i)$ denote the true membership value, falsity membership value and indeterminacy membership value of the vertex $v_i \in V$ respectively.

ii) there exist functions $\omega_T: E \to [0,1], \omega_F: E \to [0,1]$ and $\omega_I: E \to [0,1]$ such that

$$\omega_T(v_i, v_j) \leq \min\ [\alpha_T(v_i), \alpha_T(v_j)]$$

$$\omega_F(v_i, v_j) \geq \max[\alpha_F(v_i), \alpha_F(v_j)]$$
$$\omega_I(v_i, v_j) \geq \max[\alpha_I(v_i), \alpha_I(v_j)]$$
$$\text{and } 0 \leq \omega_T(v_i, v_j) + \omega_F(v_i, v_j) + \omega_I(v_i, v_j) \leq 3 \text{ for all } (v_i, v_j) \in E$$

where $\omega_T(v_i, v_j)$, $\omega_F(v_i, v_j)$, $\omega_I(v_i, v_j)$ denote the degree of true membership (DTM), degree of falsity membership (DFM) and degree of indeterminacy membership (DIM) of the edge $(v_i, v_j) \in E$, respectively.

There are various studies of graphs [16,17] and competition graphs [18,19] on neutrosophic environments. These motivate us to introduce the study on mixed graphs (MG) and competitions under the neutrosophic environments.

Since there are many instances in the current world that indicating the mixed directions of the network. For example, the current Facebook setting [20] has mixed direction. It can be noted that two friends on Facebook may be connected without following themselves. The research of mixed graphs [21] was originated in 1970. Sotskov et al. in 1976 argued the coloring of mixed graphs [22]. Liu et al. in 2015 advanced Hermitian-adjacency matrices of mixed graphs [23]. Adiga et al. in 2016 analyzed an adjacency matrix [1] of mixed graphs. Mohammed in 2017 investigated mixed graph design and isomorphism [25]. Thus there all studies on crisp environments. This limitation motivates us to define mixed graphs on neutrosophic environments and to study on competitions on mixed graphs under this environment.

Thus this study has the following major contributions.

2. Major contributions of the study

- The neutrosophic mixed graphs are presented as a generalization of mixed graphs.
- The competitions on mixed graphs under neutrosophic environment are studied with algorithms.
- The m-step competitions on mixed graphs under neutrosophic environment are studied with algorithms.
- An application to economic competition on the representation of competitions on neutrosophic mixed graphs is presented.

3. Neutrosophic mixed graphs (NMG)

Definition 3.1: *A graph $G = (V, E_1, \vec{E}_2)$ where $E_1, \vec{E}_2 \subseteq V \times V$ is neutrosophic mixed graph (NMG) if there occur functions $\rho_T: V \to [0,1], \rho_F: V \to [0,1]$ and $\rho_I: V \to [0,1]$ such that*

$$0 \leq \rho_T(u_i) + \rho_F(u_i) + \rho_I(u_i) \leq 3 \; for \; all \; u_i \in V \; (i = 1,2,3, \dots, n)$$

where $\rho_T(u_i), \rho_F(u_i), \rho_I(u_i)$ denote the DTM, DFM and DIM of the vertex $u_i \in V$ respectively and there occur functions $\mu_T: E_1 \to [0,1], \mu_F: E_1 \to [0,1], \mu_I: E_1 \to [0,1], \gamma_T: \vec{E}_2 \to [0,1], \gamma_F: \vec{E}_2 \to [0,1]$ and $\gamma_I: \vec{E}_2 \to [0,1]$ such that

$$\mu_T(v_i, v_j) \leq \min\,[\rho_T(v_i), \rho_T(v_j)]\,,\; \gamma_T(\overrightarrow{w_i, w_j}) \leq \min\,[\rho_T(w_i), \rho_T(w_j)]$$
$$\mu_F(v_i, v_j) \geq max[\rho_F(v_i), \rho_F(v_j)],\; \gamma_F(\overrightarrow{w_i, w_j}) \geq max[\rho_F(w_i), \rho_F(w_j)]$$
$$\mu_I(v_i, v_j) \geq max[\rho_I(v_i), \rho_I(v_j)],\; \gamma_I(\overrightarrow{w_i, w_j}) \geq max[\rho_I(w_i), \rho_I(w_j)]$$
$$\text{and } 0 \leq \mu_T(v_i, v_j) + \mu_F(v_i, v_j) + \mu_I(v_i, v_j) \leq 3 \text{ for all } (v_i, v_j) \in E_1$$
$$0 \leq \gamma_T(\overrightarrow{w_i, w_j}) + \gamma_F(\overrightarrow{w_i, w_j}) + \gamma_I(\overrightarrow{w_i, w_j}) \leq 3 \text{ for all } (\overrightarrow{w_i, w_j}) \in \vec{E}_2$$

where $\mu_T(v_i, v_j)$, $\mu_F(v_i, v_j)$, $\mu_I(v_i, v_j)$, $\gamma_T(v_i, v_j)$, $\gamma_F(v_i, v_j)$, $\gamma_I(v_i, v_j)$ denote the *DTM, DFM and DIM* of the edge $(v_i, v_j) \in E_1$, $(\overrightarrow{w_l, w_j}) \in \vec{E}_2$ respectively.

Example 3.2: Consider a graph with four vertices

$(v_1, (0.8, 0.5, 0.2)), (v_2, (0.9, 0.4, 0.5)), (v_3, (0.7, 0.4, 0.3)), (v_4, (0.6, 0.2, 0.3))$ and five edges $((\overrightarrow{v_1, v_2}), (0.7, 0.6, 0.5)), ((v_1, v_3), (0.6, 0.6, 0.4)), ((\overrightarrow{v_2, v_3}), (0.6, 0.5, 0.5)), ((v_1, v_4), (0.5, 0.5, 0.4)), ((\overrightarrow{v_4, v_3}), (0.5, 0.5, 0.3))$.

Clearly, the graph (Figure 1) all conditions of a neutrosophic mixed graph. Hence it is a neutrosophic mixed graph.

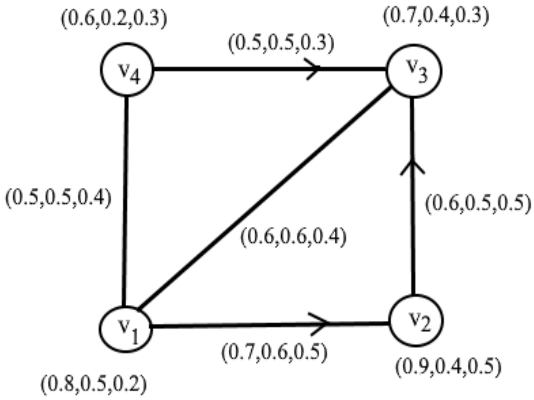

Figure 1: A neutrosophic mixed graph

Definition 3.3: *Degree is defined as triplet* $D(u) = (d(u), d^+(u), d^-(u))$ *where* $d(u)$ *is the sum of membership values of all incident edges of* E_1, $d^+(u)$ *is the sum of membership values of out-directed edges of* $\overrightarrow{E_2}$ *from the vertex* u *and* $d^-(u)$ *is the sum of membership values of in-directed edges of* $\overrightarrow{E_2}$ *in the direction to the vertex* u *where*

$$d(u) = (\textstyle\sum_v \mu_T(u, v), \sum_v \mu_F(u, v), \sum_v \mu_I(u, v))$$
$$d^+(u) = (\textstyle\sum_v \gamma_T(\overrightarrow{u, v}), \sum_v \gamma_F(\overrightarrow{u, v}), \sum_v \gamma_I(\overrightarrow{u, v}))$$
$$d^-(u) = (\textstyle\sum_v \gamma_T(\overleftarrow{u, v}), \sum_v \gamma_F(\overleftarrow{u, v}), \sum_v \gamma_I(\overleftarrow{u, v})).$$

Example 3.4: Consider a neutrosophic mixed graph (Figure 1). The degree of the vertices are $D(v_1) = ((1.1, 1.1, 0.8), (0, 0, 0), (0, 0, 0))$, $D(v_2) = ((0, 0, 0), (0.7, 0.6, 0.5), (0.6, 0.5, 0.5))$, $D(v_3) = ((0.6, 0.6, 0.4), (1.1, 1, 0.8), (0, 0, 0))$, $D(v_4) = ((0.5, 0.5, 0.4), (0, 0, 0), (0.5, 0.5, 0.3))$.

Definition 3.5: *A undirected walk in a NMG* $G = (V, E_1, \overrightarrow{E_2})$ *is a sequence:* $v_1 e_1 v_2 e_2 v_3 \ldots \ldots e_k v_{k+1}$ *where* $v_1, v_2, v_3, \ldots \ldots, v_{k+1}$ *are the vertices and* $e_1, e_2, \ldots \ldots, e_k$ *are edges respectively such that all the edges are undirected and* $\mu_T(e_i)$, $\mu_F(e_i)$, $\mu_I(e_i)$ *not all zero for* $i = 1, 2, .., k$. *An undirected walk from vertex* u *to* v *is said to be an undirected path of length* m *if there occur exactly* m *links in the walk and no vertices, no links repeat except* u, v *and it is symbolized by* $P^m_{(u,v)}$.

Definition 3.6: *A directed walk in a NMG* $G = (V, E_1, \overrightarrow{E_2}, \mu_1, \mu_2, \sigma, \delta)$ *is a sequence:* $v_1 \overrightarrow{e_1} v_2 \overrightarrow{e_2} v_3 \ldots \ldots \overrightarrow{e_k} v_{k+1}$ *where* $v_1, v_2, v_3, \ldots \ldots, v_{k+1}$ *are the vertices and* $\overrightarrow{e_1}, \overrightarrow{e_2}, \ldots \ldots, \overrightarrow{e_k}$ *are edges respectively such that all the edges are directed with the same direction and* $\mu_T(\overrightarrow{e_i})$, $\mu_F(\overrightarrow{e_i})$, $\mu_I(\overrightarrow{e_i})$ *not all zero for,* $i = 1, 2, .., k$. *A directed walk from vertex* u *to* v *is said to be a directed path of length* m *if there occur exactly* m *links in the walk and no vertices, no links repeat except* u, v *and it is symbolized by* $\vec{P}^m_{(u,v)}$.

Example 3.7: Consider a NMG (Figure 1). Here $v_3 - v_1 - v_4$ is an undirected path and $v_1 \rightarrow v_2 \rightarrow v_3$ is a directed path.

Definition 3.8: *Let $G = (V, E_1, \vec{E}_2)$ be a NMG. Then the neighbourhood $N(v_i)$, out-neighbourhood $N^+(v_i)$ and in-neighbourhood $N^-(v_i)$ of a vertex $v_i \in V$ is given by*

$$N(v_i) = \{v_j, (\mu_T(v_i, v_j), \mu_F(v_i, v_j), \mu_I(v_i, v_j)) : (v_i, v_j) \in E_1\}$$
$$N^+(v_i) = \{v_j, (\gamma_T(\overrightarrow{v_i, v_j}), \gamma_F(\overrightarrow{v_i, v_j}), \gamma_I(\overrightarrow{v_i, v_j})) : (\overrightarrow{v_i, v_j}) \in \vec{E}_2\}$$
$$N^+(v_i) = \{v_j, (\gamma_T(\overrightarrow{v_i, v_j}), \gamma_F(\overrightarrow{v_i, v_j}), \gamma_I(\overrightarrow{v_i, v_j})) : (\overrightarrow{v_i, v_j}) \in \vec{E}_2\}$$

where $\gamma_T(\overrightarrow{v_i, v_j}), \gamma_F(\overrightarrow{v_i, v_j}), \gamma_I(\overrightarrow{v_i, v_j})$ denote the DTM, DFM and DIM of edge $(\overrightarrow{v_i, v_j}) \in \vec{E}_2$.

Example 3.9: Consider a neutrosophic mixed graph (Figure 2). Then $N^+(v_1)$ and $N^-(v_3)$ are given by

$$N(v_1) = \{v_4, (06, 0.3, 04)\}$$
$$N^+(v_1) = \{v_2, (0.5, 0.4, 0.3)\}\}$$
$$N^-(v_3) = \{v_2, (0.5, 0.4, 0.4)\}$$

Definition 3.10: *Let $G = (V, E_1, \vec{E}_2)$ be a neutrosophic mixed graph. Then the $m-step$ neighbourhood $N_m(v_i)$, out-neighbourhood $N_m^+(v_i)$, in-neighbourhood $N_m^-(v_i)$ of a vertex $v_i \in V$ is given by*

$$N_m(v_i) = \left\{\left(v_j, (\mu_T^m(v_i, v_j), \mu_F^m(v_i, v_j), \mu_I^m(v_i, v_j)\right) : v_j \in V\right\} \textit{ where}$$
$$\mu_T^m(v_i, v_j) = min\left\{\mu_T(a, b) : \forall (a, b) \in P_{(v_i, v_j)}^m\right\},$$
$$\mu_F^m(v_i, v_j) = max\left\{\mu_F(a, b) : \forall (a, b) \in P_{(v_i, v_j)}^m\right\},$$
$$\mu_I^m(v_i, v_j) = max\left\{\mu_I(a, b) : \forall (a, b) \in P_{(v_i, v_j)}^m\right\},$$

$\mu_T(a, b), \mu_F(a, b)$ and $\mu_I(a, b)$ denote DTM, DFM and DIM of edge $(a, b) \in E_1$.

$$N_m^+(v_i) = \left\{\left(v_j, (\gamma_T^m(v_i, v_j), \gamma_F^m(v_i, v_j), \gamma_I^m(v_i, v_j)\right) : v_j \in V\right\} \textit{ where}$$
$$\gamma_T^m(v_i, v_j) = \min\left\{\gamma_T(\overrightarrow{a, b}) : \forall (\overrightarrow{a, b}) \in \vec{P}_{(v_i, v_j)}^m\right\}$$
$$\gamma_F^m(v_i, v_j) = \max\left\{\gamma_F(\overrightarrow{a, b}) : \forall (\overrightarrow{a, b}) \in \vec{P}_{(v_i, v_j)}^m\right\}$$
$$\gamma_T^m(v_i, v_j) = \max\left\{\gamma_T(\overrightarrow{a, b}) : \forall (\overrightarrow{a, b}) \in \vec{P}_{(v_i, v_j)}^m\right\}$$
$$N_m^-(v_i) = \left\{\left(v_j, (\gamma_T^m(v_i, v_j), \gamma_F^m(v_i, v_j), \gamma_I^m(v_i, v_j)\right) : v_j \in V\right\} \text{ where}$$
$$\gamma_T^m(v_i, v_j) = \min\left\{\gamma_T(\overrightarrow{a, b}) : \forall (\overrightarrow{a, b}) \in \vec{P}_{(v_j, v_i)}^m\right\}$$
$$\gamma_F^m(v_i, v_j) = \max\left\{\gamma_F(\overrightarrow{a, b}) : \forall (\overrightarrow{a, b}) \in \vec{P}_{(v_j, v_i)}^m\right\}$$
$$\gamma_T^m(v_i, v_j) = \max\left\{\gamma_T(\overrightarrow{a, b}) : \forall (\overrightarrow{a, b}) \in \vec{P}_{(v_j, v_i)}^m\right\}$$

where $\gamma_T(\overrightarrow{a, b}), \gamma_F(\overrightarrow{a, b}), \gamma_I(\overrightarrow{a, b})$ denote *DTM, DFM and DIM* of edge $(\overrightarrow{a, b}) \in \vec{E}_2$.

Example 3.11: Consider a neutrosophic mixed graph (Figure 2). Then $N_2^+(v_1)$ and $N_2^-(v_3)$ are given by

$$N(v_1) = \{v_3, (0.6, 0.4, 0.4)\}, N_2^+(v_1) = \{v_3, (0.5, 0.4, 0.4)\} = N_2^-(v_3)$$

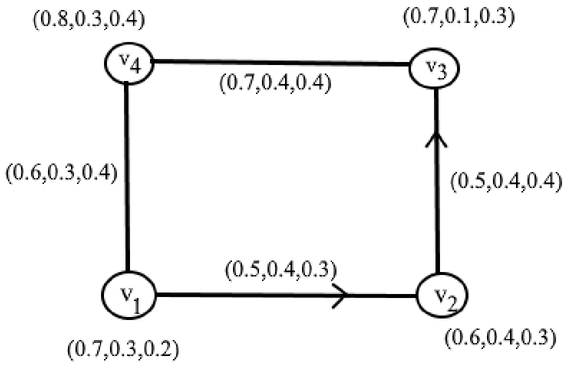

Figure 2: A neutrosophic mixed graph

Definition 3.12: The $m-$ step NMG $G_m = (V^m, E_1^m, E_2^m)$ of a NMG $G = (V, E_1, \overrightarrow{E_2})$ is defined as follows:

Vertex set of G_m is $V^m = V$, Edge set of G_m is $E_1^m = \{(u_i, u_j): v_j \in N_m(u_j)\}$, $\vec{E}_2^m = \{v: v \in N_m^+(v_i) \cup N_m^-(v_i)\}$. The degree of membership values of vertex follows from Definitions 3.8 and 3.10.

4. Competitions on NMG

Definition 4.1: *Let $G = (V, E_1, \vec{E}_2)$ be a NMG. Then the neutrosophic competition graph $C(G) = (V, E)$ of $G = (V, E_1, \vec{E}_2)$ is a neutrosophic graph which has the equal vertex set V and neutrosophic edge occur between u, v iff $N^+(u) \cap N^+(v) \neq \emptyset$ such that*

$$\varphi_T(u,v) = \min_{w \in N^+(u) \cap N^+(v)} |\gamma_T(\overrightarrow{u,w}) - \gamma_T(\overrightarrow{v,w})|$$
$$\varphi_F(u,v) = \max_{w \in N^+(u) \cap N^+(v)} |\gamma_F(\overrightarrow{u,w}) - \gamma_F(\overrightarrow{v,w})|$$
$$\varphi_I(u,v) = \max_{w \in N^+(u) \cap N^+(v)} |\gamma_I(\overrightarrow{u,w}) - \gamma_I(\overrightarrow{v,w})|$$

where $\varphi_T(u,v), \varphi_F(u,v), \varphi_I(u,v)$ denote the DTM, DFM and DIM of edge $(u,v) \in E$.

Algorithm 4.2: *To find the degree of memberships of edges of a neutrosophic competition graph for a NMG and its matrix representation.*

Step-1: Let us consider a neutrosophic mixed graph with n vertices.

Step-2: Find the out-neighbourhood $N^+(u_i)$ for $i = 1, 2, \ldots, n$ if there exist edges $(\overrightarrow{u_i, u_k})$ for $u_i, u_k \in V$.

Step-3: Find the sets $N^+(u_i) \cap N^+(u_j)$ for each pair of vertices $u_i\ u_j \in V$.

Step-4: If $N^+(u_i) \cap N^+(u_j) \neq \Phi$ for $u_i, u_j \in V$ and $N^+(u_i) \cap N^+(u_j) = \{u_p, p = 1,2, \ldots, \neq i, j\}, say,$ then find $\gamma_T(\overrightarrow{u_i, u_p})$, $\gamma_T(\overrightarrow{u_j, u_p})$; $\gamma_F(\overrightarrow{u_i, u_p}), \gamma_F(\overrightarrow{u_j, u_p})$; $\gamma_I(\overrightarrow{u_i, u_p})$, $\gamma_I(\overrightarrow{u_j, u_p})$ for $p = 1,2, \ldots, \neq i, j$.

Step-5: Find $|\gamma_T(\overrightarrow{u_i, u_p}) - \gamma_T(\overrightarrow{u_j, u_p})|$, $|\gamma_F(\overrightarrow{u_i, u_p}) - \gamma_F(\overrightarrow{u_j, u_p})|$, $|\gamma_I(\overrightarrow{u_i, u_p}) - \gamma_I(\overrightarrow{u_j, u_p})|$ for $p = 1,2, \ldots, \neq i, j$.

Step-6: Find $\varphi_T(u_i, u_j) = min\{|\gamma_T(\overrightarrow{u_i, u_p}) - \gamma_T(\overrightarrow{u_j, u_p})|, p = 1,2, \ldots, \neq i, j\}$

$$\varphi_F(u_i, u_j) = max\{|\gamma_F(\overrightarrow{u_i, u_p}) - \gamma_F(\overrightarrow{u_j, u_p})|, p = 1,2, \ldots, \neq i, j\}$$
$$\varphi_I(u_i, u_j) = max\{|\gamma_I(\overrightarrow{u_i, u_p}) - \gamma_I(\overrightarrow{u_j, u_p})|, p = 1,2, \ldots, \neq i, j\}$$

Step-6: The matrix of competition neutrosophic graph is a square matrix. Its order same to the number of vertices. It is defined as below.

$$a_{ij} = \begin{cases} (\varphi_T(u_i, u_j), \varphi_F(u_i, u_j), \varphi_I(u_i, u_j)) & if\ there\ is\ an\ edge\ between\ u_i and\ u_j. \\ (0,0,0), & if\ there\ is\ no\ edge\ between\ u_i and\ u_j. \end{cases}$$

Example 4.3: Consider a neutrosophic mixed house graph (Figure 3).

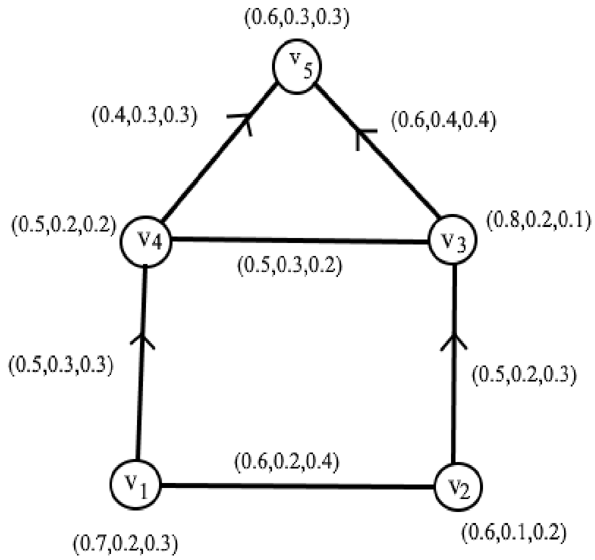

Figure 3: A neutrosophic mixed graph

$N^+(v_1) = \{v_4, (0.5,0.3,0.3)\}$, $N^+(v_2) = \{v_3, (0.5,0.2,0.3)\}$, $N^+(v_3) = \{v_5, (0.6,0.4,0.4)\}$, $N^+(v_4) = \{v_5, (0.4,0.3,0.3)\}$, $N^+(v_5) = \emptyset$.

Hence $N^+(v_3) \cap N^+(v_4) \neq \emptyset$ only and there exist an edge between vertex v_3 and v_4 in the corresponding competition neutrosophic graph where $\varphi_T(v_3, v_4) = 0.2$, $\varphi_F(v_3, v_4) = 0.1$ and $\varphi_I(v_3, v_4) = 0.1$

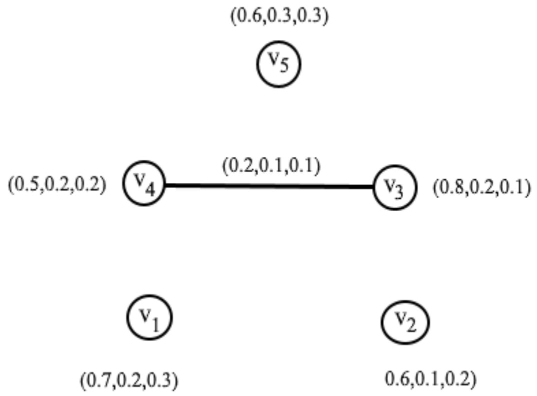

Figure 4: Competition neutrosophic graph of Figure 3

Definition 4.4: *Let $G = (V, E_1, \vec{E}_2)$ be a NMG. Then the $m-$ step NMG $C^m(G) = (V, E)$ of a NMG $G = (V, E_1, \vec{E}_2)$ is a neutrosophic graph which has the equal vertex set V and also neutrosophic edge occur between u, v iff $N_m^+(u) \cap N_m^+(v) \neq \emptyset$ such that*

$$\varphi_T^m(u,v) = \min_{w \in N_m^+(u) \cap N_m^+(v)} |\gamma_T^m(\overrightarrow{u,w}) - \gamma_T^m(\overrightarrow{v,w})|$$
$$\varphi_F^m(u,v) = \max_{w \in N_m^+(u) \cap N_m^+(v)} |\gamma_F^m(\overrightarrow{u,w}) - \gamma_F^m(\overrightarrow{v,w})|$$
$$\varphi_I^m(u,v) = \max_{w \in N^+(u) \cap N^+(v)} |\gamma_I^m(\overrightarrow{u,w}) - \gamma_I^m(\overrightarrow{v,w})|$$

where $\varphi_T^m(u,v), \varphi_F^m(u,v), \varphi_I^m(u,v)$ denote DTM, DFM and DIM of edge $(u,v) \in E$.

Algorithm 4.5: *To find the membership values of edges of a $m-$step competition neutrosophic graph for a NMG and its matrix representation.*

Step-1: Let us consider a neutrosophic mixed graph with n vertices.

Step-2: Find the $m-$ step out-neighbourhood $N_m^+(u_i)$ for $i = 1, 2, \ldots, n$ if there exist edges $(\overrightarrow{u_i, u_k})$ for $u_i, u_k \in V$.

Step-3: Find the sets $N_m^+(u_i) \cap N_m^+(u_j)$ for each pair of vertices $u_i\ u_j \in V$.

Step-4: If $N_m^+(u_i) \cap N_m^+(u_j) \neq \Phi$ for $u_i, u_j \in V$ and $N_m^+(u_i) \cap N_m^+(u_j) = \{u_p, p = 1,2, \ldots, \neq i, j\}, say,$ then find $\gamma_T^m(\overrightarrow{u_i, u_p})$, $\gamma_T^m(\overrightarrow{u_j, u_p})$; $\gamma_F^m(\overrightarrow{u_i, u_p}), \gamma_F^m(\overrightarrow{u_j, u_p})$; $\gamma_I^m(\overrightarrow{u_i, u_p}), \gamma_I^m(\overrightarrow{u_j, u_p})$ for $p = 1,2, \ldots, \neq i, j$.

Step-5: Find $|\gamma_T^m(\overrightarrow{u_i, u_p}) - \gamma_T^m(\overrightarrow{u_j, u_p})|$, $|\gamma_F^m(\overrightarrow{u_i, u_p}) - \gamma_F^m(\overrightarrow{u_j, u_p})|$, $|\gamma_I^m(\overrightarrow{u_i, u_p}) - \gamma_I^m(\overrightarrow{u_j, u_p})|$ for $m = 1,2, \ldots, \neq i, j$.

Step-6: Find $\varphi_T^m(u_i, u_j) = min\{|\gamma_T^m(\overrightarrow{u_i, u_p}) - \gamma_T^m(\overrightarrow{u_j, u_p})|, p = 1,2, \ldots, \neq i, j\}$

$$\varphi_F^m(u_i, u_j) = max\{|\gamma_F^m(\overrightarrow{u_i, u_p}) - \gamma_F^m(\overrightarrow{u_j, u_p})|, p = 1,2, \ldots, \neq i, j\}$$
$$\varphi_I^m(u_i, u_j) = max\{|\gamma_I^m(\overrightarrow{u_i, u_p}) - \gamma_I^m(\overrightarrow{u_j, u_p})|, p = 1,2, \ldots, \neq i, j\}$$

Step-6: **:** The matrix of competition neutrosophic graph is a square matrix. Its order equal to the number of vertices. Its entries are given below.

$$a_{ij} = \begin{cases} \left(\varphi_T^m(u_i, u_j), \varphi_F^m(u_i, u_j), \varphi_I^m(u_i, u_j)\right) & if\ there\ is\ an\ edge\ between\ u_i and\ u_j. \\ (0,0,0), & if\ there\ is\ no\ edge\ between\ u_i and\ u_j. \end{cases}$$

Example 4.6: Consider a neutrosophic mixed graph (Figure 3). Here ${N_2}^+(v_1) \cap {N_2}^+(v_2) \neq \emptyset$ only and there exist an edge between vertex v_1 and v_2 in the corresponding 2 $-$step competition neutrosophic graph with $\varphi_T^m(v_1, v_2) = 0.1, \varphi_F^m(v_1, v_2) = 0.1$ and $\varphi_I^m(v_1, v_2) = 0.1$. thus the $2-$step competition neutrosophic graph is given below.

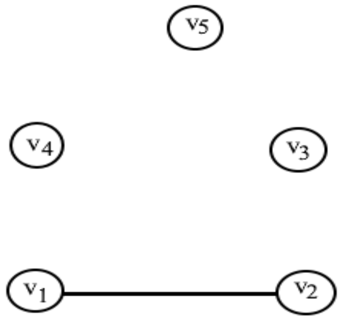

Figure 5. 2-step competition neutrosophic graph of Figure 5.

Theorem 4.7: *If G_m be the $m-$step graph of a graph G, then $C(G_m) = C_m(G)$.*

Proof. Let G be a NMG and G_m be its $m - step$ NMG. Obviously, the vertex set of two graphs are the equal. Let $(u, v) \in C(G_m)$. Then there exist edges $(\overrightarrow{u, y_1})$, $(\overrightarrow{v, y_1})$; $(\overrightarrow{u, y_2})$, $(\overrightarrow{v, y_2})$;……, $(\overrightarrow{u, y_n})$, $(\overrightarrow{v, y_n})$ for some integer n. Now, $N^+(u) \cap N^+(v) = \{y_1, y_2,……., y_n\}$. Since an edge $(\overrightarrow{u, y_1}) \in G_m$ suggests there occur a path of length m from u to y_1 and similar for $(\overrightarrow{v, y_1}) \in G_m$. Hence $(u, v) \in C_m(G)$. Similarly, if an edge $(x, y) \in C_m(G)$ then implies $(x, y) \in C(G_m)$. Therefore $C(G_m) = C_m(G)$.

Note 4.8: *In the fuzzy mixed graph, if a vertex has only in-neighbours, then the vertex must be isolated in the corresponding competition graph.*

Note 4.9: *Competition graph on every mixed circuit graph is a null graph.*

5. Application to economic competition

Like competitions in the ecosystem, economic competition among some countries is considered here for representations. The competitions among countries with respect to GDP (Gross Domestic Product), GPI (Global Peace Index) and DRI (Disaster Risk Index) of all countries are evaluated. So we need to construct a network before evaluation and presentation.

5.1 Construction of a network

To construct a network, we consider the twelve countries which are given in figure 8 with their respective national flag.

All countries are connected as they are competing for Gross Domestic Product (GDP) (Figure 6). So we take undirected links among countries and directed links to GDP from all countries. Here GDP, GPI and DRI data are collected from Wikipedia and presented in Table 1 with normalized form. In this case, the normalized value of GDP value, GPI value and DR is taken as *DTM, DFM and DIM* respectively.

Figure 8: Economic competition among twelve countries

SL NO	Country	GDP,2019 Million	Normalized GDP	GPI,2019	Normalized GPI	DRI, 2017	Normalized DRI
1	Germany	38,63,344	1	1.547	0.5	2.95	0.444
2	India	29,35,570	0.76	2.605	0.842	6.64	1
3	United Kingdom	27,43,586	0.71	1.801	0.582	3.54	0.533
4	France	27,07,074	0.701	1.892	0.612	2.62	0.395
5	Italy	19,88,636	0.515	1.754	0.567	4.42	0.666
6	Brazil	18,47,020	0.478	2.271	0.734	4.09	0.616
7	Canada	17,30,914	0.448	1.327	0.429	3.01	0.453
8	Russia	16,37,892	0.424	3.093	1	3.58	0.539
9	South Korea	16,29,532	0.422	1.867	0.604	4.59	0.691
10	Spain	13,97,870	0.362	1.699	0.549	3.05	0.459
11	Australia	13,76,255	0.356	1.419	0.459	4.22	0.636
12	Mexico	12,74,175	0.33	2.6	0.841	5.97	0.899

Table 1: Collection of data of countries from Wikipedia on 8th May, 2020.

5.2 Algorithms

The result of competitions among countries to GDP growth is deduced from the following steps.

Step-1: Consider a network among countries and GDP. All vertices and degrees of membership are taken as input.

Step-2: Find the out-neighbours of the countries and the intersections of all pairwise out-neighbours of the countries.

Step-4: Find the degree of memberships for competing countries towards GDP using Table 1.

Step-5: The required result for the competition is obtained using algorithm.

Step-6: Define ranking function $R = T \times \frac{1+F+I}{3}$ and find the value of R to obtain competition value.

5.3 Results

The membership values of corresponding competition graph are given in the following three Tables. Table 2 represents the DTM, Table 3 represents the DFM and Table 4 represents the DIM of the competition neutrosophic mixed graph (CNMG). The ranking of competitions is represented in Table 5. The minimum of the value of R will indicate that the countries have more competition.

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.24	0.29	0.299	0.485	0.522	0.552	0.576	0.578	0.638	0.644	0.67
2	0.24	0	0.05	0.059	0.245	0.282	0.312	0.336	0.338	0.398	0.404	0.43
3	0.29	0.05	0	0.009	0.195	0.232	0.262	0.286	0.288	0.348	0.354	0.38
4	0.299	0.059	0.009	0	0.186	0.223	0.253	0.277	0.279	0.339	0.345	0.371
5	0.485	0.245	0.195	0.186	0	0.037	0.067	0.091	0.093	0.153	0.159	0.185
6	0.522	0.282	0.232	0.223	0.037	0	0.03	0.054	0.056	0.116	0.122	0.148
7	0.552	0.312	0.262	0.253	0.067	0.03	0	0.024	0.026	0.086	0.092	0.118
8	0.576	0.336	0.286	0.277	0.091	0.054	0.024	0	0.002	0.062	0.068	0.094
9	0.578	0.338	0.288	0.279	0.093	0.056	0.026	0.002	0	0.06	0.066	0.092
10	0.638	0.398	0.348	0.339	0.153	0.116	0.086	0.062	0.06	0	0.006	0.032
11	0.644	0.404	0.354	0.345	0.159	0.122	0.092	0.068	0.066	0.006	0	0.026
12	0.67	0.43	0.38	0.371	0.185	0.148	0.118	0.094	0.092	0.032	0.026	0

Table 2: Degree of truth membership values of CNMG

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.342	0.082	0.112	0.067	0.234	0.071	0.5	0.104	0.049	0.041	0.341
2	0.342	0	0.26	0.23	0.275	0.108	0.413	0.158	0.238	0.293	0.383	0.001
3	0.082	0.26	0	0.03	0.015	0.152	0.153	0.418	0.022	0.033	0.123	0.259
4	0.112	0.23	0.03	0	0.045	0.122	0.183	0.388	0.008	0.063	0.153	0.229
5	0.067	0.275	0.015	0.045	0	0.167	0.138	0.433	0.037	0.018	0.108	0.274
6	0.234	0.108	0.152	0.122	0.167	0	0.305	0.266	0.13	0.185	0.275	0.107
7	0.071	0.413	0.153	0.183	0.138	0.305	0	0.571	0.175	0.12	0.03	0.412
8	0.5	0.158	0.418	0.388	0.433	0.266	0.571	0	0.396	0.451	0.541	0.159
9	0.104	0.238	0.022	0.008	0.037	0.13	0.175	0.396	0	0.055	0.145	0.237
10	0.049	0.293	0.033	0.063	0.018	0.185	0.12	0.451	0.055	0	0.09	0.292
11	0.041	0.383	0.123	0.153	0.108	0.275	0.03	0.541	0.145	0.09	0	0.382
12	0.341	0.001	0.259	0.229	0.274	0.107	0.412	0.159	0.237	0.292	0.382	0

Table 3: Degree of falsity membership values of CNMG.

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.556	0.089	0.049	0.222	0.172	0.009	0.095	0.247	0.015	0.192	0.455
2	0.556	0	0.467	0.605	0.334	0.384	0.547	0.461	0.309	0.541	0.364	0.101
3	0.089	0.467	0	0.138	0.133	0.083	0.08	0.006	0.158	0.074	0.103	0.366

4	0.049	0.605	0.138	0	0.271	0.221	0.058	0.144	0.296	0.064	0.241	0.504
5	0.222	0.334	0.133	0.271	0	0.05	0.213	0.127	0.025	0.207	0.03	0.233
6	0.172	0.384	0.083	0.221	0.05	0	0.163	0.077	0.075	0.157	0.02	0.283
7	0.009	0.547	0.08	0.058	0.213	0.163	0	0.086	0.238	0.006	0.183	0.446
8	0.095	0.461	0.006	0.144	0.127	0.077	0.086	0	0.152	0.08	0.097	0.36
9	0.247	0.309	0.158	0.296	0.025	0.075	0.238	0.152	0	0.232	0.055	0.208
10	0.015	0.541	0.074	0.064	0.207	0.157	0.006	0.08	0.232	0	0.177	0.44
11	0.192	0.364	0.103	0.241	0.03	0.02	0.183	0.097	0.055	0.177	0	0.263
12	0.455	0.101	0.366	0.504	0.233	0.283	0.446	0.36	0.208	0.44	0.263	0

Table 4: Degree of indeterminacy membership values of CNMG.

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0.152	0.113	0.116	0.208	0.245	0.199	0.306	0.26	0.226	0.265	0.401
2	0.152	0	0.029	0.036	0.131	0.14	0.204	0.181	0.174	0.243	0.235	0.158
3	0.113	0.029	0	0.004	0.075	0.096	0.108	0.136	0.113	0.128	0.145	0.206
4	0.116	0.036	0.004	0	0.082	0.1	0.105	0.141	0.121	0.127	0.16	0.214
5	0.208	0.131	0.075	0.082	0	0.015	0.03	0.047	0.033	0.062	0.06	0.093
6	0.245	0.14	0.096	0.1	0.015	0	0.015	0.024	0.022	0.052	0.053	0.069
7	0.199	0.204	0.108	0.105	0.03	0.015	0	0.013	0.012	0.032	0.037	0.073
8	0.306	0.181	0.136	0.141	0.047	0.024	0.013	0	0.001	0.032	0.037	0.048
9	0.26	0.174	0.113	0.121	0.033	0.022	0.012	0.001	0	0.026	0.026	0.044
10	0.226	0.243	0.128	0.127	0.062	0.052	0.032	0.032	0.026	0	0.003	0.018
11	0.265	0.235	0.145	0.16	0.06	0.053	0.037	0.037	0.026	0.003	0	0.014
12	0.401	0.158	0.206	0.214	0.093	0.069	0.073	0.048	0.044	0.018	0.014	0

Table 5: Competition values of countries after ranking

Thus a real life competition to economy is presented using the concepts of proposed algorithms. We observed the following items from the results.

- The non-zero lower value in the Table 5 indicate that the competition among the corresponding countries are higher.
- The competition value should be use as the compareness between two items.
- The study of competition should be apply as model for data analysis in marketing.

6. Conclusions

In this article, we introduced the neutrosophic mixed graphs as a generalization from mixed graphs, competitions on neutrosophic mixed graphs, and competitions number. Also the real life presentation of competition on neutrosophic mixed graphs has been illustrated. Thus this knowledge will be extremely effective for future research to advance the most of NMG theory topics i.e. interval-valued neutrosophic mixed graphs, generalized neutrosophic mixed graphs, generalized neutrosophic mixed graphs, neutrosophic planar graphs etc. and relevant in numerous real life difficulties.

References

[1] Samanta, S., Pal, M.: Fuzzy planar graphs. IEEE Trans. Fuzzy Syst. 23(6), 1936–1942 (2015).
[2] J. E. Cohen, Interval graphs and food webs: a finding and a problem, Document 17696-PR, RAND Corporation, Santa Monica, CA, 1968.
[3] F. S. Roberts, Discrete Mathematical Models, with Applications to Social, Biological, and

Environmental Problems, Prentice-Hall, Englewood Cliffs, NJ, 1976.

[4] F. S. Roberts, Food webs, competition graphs, and the boxicity of ecological phase space, in Theory and Applications of Graphs, (Y. Alavi and D. Lick, eds.), Springer-Verlag, New York, 477–490, 1978.

[5] R. J. Opsut, On the computation of the competition number of a graph, SIAM Journal on Algebraic Discrete Mathematics, 3, 420-428, 1982.

[6] S. R. Kim, T. A. McKee, R. R. McMorris and F. S. Roberts, p-competition number, Discrete Applied Mathematics, 46, 89-92, 1993.

[7] S. R. Kim, T. A. McKee, R. R. McMorris and F. S. Roberts, p-competition graph, Linear Algebra and its Applications, 217, 167-178, 1995.

[8] K. A. S. Factor and S. K. Merz, The (1,2) –step competition graph of a tournament, Discrete Applied mathematics, 159, 100-103, 2011.

[9] L. A. Zadeh, Fuzzy sets, Information and Control, **8**, 338–353, 1965.

[10] A. Kauffman, Introduction a la Theorie des Sous-emsembles Flous, Paris: Masson et Cie Editeurs, 1973.

[11] S. Samanta and M. Pal, Fuzzy k-competition graphs and p-competition fuzzy graphs, Fuzzy Information and Engineering, 5, 191-204, 2013.

[12] S. Samanta, B. Sarkar, D. Shin and M. Pal, Completeness and regularity of generalized fuzzy graphs, Springer Plus, 5, 2016.

[13] S. Samanta and B. Sarkar, Representation of competitions by generalized fuzzy graphs, International Journal of Computational Intelligence System, 11, 1005-1015, 2018.

[14] F. Smarandache, Neutrosophy neutrosophic probability, Set and Logic, Amer Res Press, Rehoboth, USA, 1998.

[15] S. Broumi, M. Talea, A. Bakali and F. Smarandache, Single valued neutrosophic graphs, Journal of New Theory, 10, 86-101, 2016.

[16] M. Akram and G. Shahzadi, Operations on single-valued neutrosophic graphs, Journal of Uncertain Systems,11(1), 1-26, 2017.

[17] S. Broumi, A. Bakali, M. Talea, F. Smarandache and A. Hassan, Generalized single-valued neutrosophic graphs of first type, Acta Electrotechica, 59, 23-31, 2018.

[18] M. Akram and S. Siddique, Neutrosophic competition graphs with applications, Journal Intelligence and Applications, 33, 921-935, 2017.

[19] K. Das, S. Samanta and K. De, Generalized Neutrosophic Competition Graphs, Neutrosophic Sets and Systems, 31, 156-171**,** 2020.

[20] C. Yang & Y. Lee, Interactants and activities on Facebook, Instagram, and Twitter: Associations between social media use and social adjustment to college, Applied Developmental Science, 24, 1, 62-78, 2020.

[21] N. V. Lambin and V. S. Tanaev, On a circuit-free orientation of mixed graphs, Dokladi Akademii Navuk BSSR, 14, 780–781, 1970.

[22] Y. N. Sotskov and V.S. Tanaev, Chromatic polynomial of a mixed graph, Vestsi Akademii Navuk BSSR, Seriya Fiz.-Mat Navuk 6, 20–23, 1976.

[23] J. Liu, X. Li, Hermitian-adjacency matrices and Hermitian energies of mixed graphs, Linear Algebra Appl., 466, 182–207, 2015.

[24] C. Adiga, B.R. Rakshith, W. So, On the mixed adjacency matrix of a mixed graph, Linear Algebra and its Applications, 495, 223-241, 2016.

[25] A. M. Mohammed, Mixed Graph Representation and Mixed Graph Isomorphism, Journal of Science, 30(1), 303-310, 2017.

[26] R. Mahapatra, S. Samanta, M. Pal, Q. xin, RSM index: a new way of link prediction in social networks, Journal of Intelligent and Fuzzy Systems, 37, 2137-2151, 2019, DOI: 10.3233/JIFS-181452.

[27] R. Mahapatra, S. Samanta, T. Allahviranloo and M. Pal, Radio fuzzy graphs and assignment of frequency in radio stations, Computational and Applied Mathematics,38, 117, DOI: 10.1007/s40314-019-0888-3, 2019.

[28] R. Mahapatra, S. Samanta, M. Pal, Applications of Edge Colouring of Fuzzy Graphs, INFORMATICA, 31(2), 313-330, https://doi.org/10.15388/20-INFOR403, 2020.

[29] R. Mahapatra, S. Samanta, M. Pal, Generalized Neutrosophic Planar Graphs and its Application, J. Appl. Math. Comput. 65, 693–712, 2020. https://doi.org/10.1007/s12190-020-01411-x

[30] R. Mahapatra, S. Samanta, M. Pal, Q. xin, Link Prediction in Social Networks by Neutrosophic Graph, International Journal of Computational Intelligence Systems, 13(1), 1699 - 1713, 2020, https://doi.org/10.2991/ijcis.d.201015.002.

[31] R. Mahapatra, S. Samanta, M. Pal, J. Lee, S. Khan, U. Naseem, R. Bhadoria, Colouring of COVID-19 Affected Region Based on Fuzzy Directed Graphs, Computers, Materials and Continua, 68(1), 1219-1233, 2021, doi:10.32604/cmc.2021.015590.

[32] R. Mahapatra, S. Samanta, R. Bhadoria, M. Pal, T. Allahviranloo, B. Pandey, A Graph Networks Based Quality Control Model For Packaged Food Smart Traceability and Communication, European Journal of Molecular and Clinical Medicine, 7(6), 2830-2848, 2020.

[33] R. Mahapatra, S. Samanta, M. Pal, Discrete Dynamics in Nature and Society, vol. 2022, Article ID 1149724, 8 pages, 2022. https://doi.org/10.1155/2022/1149724

[34] R. Mahapatra, S. Samanta, M. Pal, Detecting influential node in a network using neutrosophic graph and its application, Soft Comput 27, 9247–9260 (2023). https://doi.org/10.1007/s00500-023-08234-5.