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Review Article

COUPLED FIXED POINT THEOREM FOR FOUR MAPS ON A METRIC SPACES ENDOWED WITH A GRAPH

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Abstract
The main aim of this paper to introduce a new notation $G-fg-$ contraction and a new edge preserving property. With help of this proved a coupled coincidence fixed point theorem for four maps with a graph in a metric space.

Keywords: Metric spaces with a graph, edge preserving, coupled fixed point.

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INTRODUCTION

In 2006, the concepts of fixed point theory and graph theory were combined by Espinola and Kirk ([3]). Jachymski([4]) and Chifu ([2]) came up with an interesting idea of using the language of graph theory in the study of fixed point results.

A graph is an ordered pair $G=(V,E)$, where V is a non empty set and the elements in V are called vertices or nodes and E is a binary relation on V. i.e., $E \subseteq (V \times T)$. The elements of E are called edges.

In this paper we concentrate on directed graphs.

Let G^{-1} be the conversion of the graph G. i.e., the graph obtained from G by reversing the direction of edges. Simply, $E(G^{-1})=\{(y,x):(x,y)\in E(G)\}$.

A directed graph G is called a oriented graph if $(x,y)\in E(G)$, then $(y,x)\notin E(G)$.

Definition 1.1 *[2] A function $S: X\times X \to X$ is said to be $G-$ continuous if $\{x_{n_i}\}\to p$, $\{y_{n_i}\}\to q$ and $(x_{n_i},x_{n_{i+1}})\in E(G)$, $(y_{n_i},y_{n_{i+1}})\in E(G^{-1})$ implies $S(x_{n_i},x_{n_{i+1}})\to S(p,q)$ and $S(y_{n_i},y_{n_{i+1}})\to S(q,p)$ as $i\to\infty$, where $(x,y),(p,q)\in X\times X$ and $(n_i)_{i\in N}$ be a sequence of positive integers.*

Definition 1.2 *[2]) Let (X,d) be a complete metric space endowed with a directed graph G. Then the triplet (X,d,G) has property (A) if*

(i) for any sequence $\{x_n\}_{n\in N}$ in X such that $\{x_n\}\to p$ and $(x_n,x_{n+1})\in E(G)$ implies $(x_n,p)\in E(G)$

(ii) for any sequence $\{y_n\}_{n\in N}$ in X such that $\{y_n\}\to q$ and $(y_n,y_{n+1})\in E(G^{-1})$ implies $(y_n,q)\in E(G^{-1})$.

Many authors studied about the coupled fixed points and coupled coincident points and common coupled fixed points and the G- continuous properties (see [2], [19], [15], [7]) By taking the inspiration from the above authors G. Adilakshmi and G.N.V. Kishore([1]) introduced a $G-fg$ contraction on metric space endowed with a graph for four mappings.

Definition 1.3 *([1]) Suppose (X,d) be a metric space endowed with a directed graph G. Let us consider the mappings $S,T: X\times X\to X$ and $f,g: X\to X$ with defining the following sets*

(I) $(X\times X)_{Sf}=\{(x,y)\in X\times X: (fx,S(x,y))\in E(G),(fy,S(y,x))\in E(G^{-1})\}$

and (i) f is edge preserving. i.e., $(fx,fu)\in E(G),(fy,fv)\in E(G^{-1})$

implies $(f(fx),f(fu))\in E(G)$ and $(f(fy),f(fv))\in E(G^{-1})$.

(ii) S is f edge preserving. i.e., $(fx,fu)\in E(G),(fy,fv)\in E(G^{-1})$

implies $(S(x,y),S(u,v))\in E(G)$ and $(S(y,x),S(v,u))\in E(G^{-1})$.

(II) $(X\times X)_{Tg}=\{(u,v)\in X\times X: (gu,T(u,v))\in E(G),(gv,T(v,u))\in E(G^{-1})\}$

and (i) g is edge preserving. i.e., $(gx,gu)\in E(G),(gy,gv)\in E(G^{-1})$

implies $(g(gx),g(gu))\in E(G)$ and $(g(gy),g(gv))\in E(G^{-1})$.

(ii) T is g edge preserving. i.e., $(gx,gu)\in E(G),(gy,gv)\in E(G^{-1})$

implies $(T(x,y),T(u,v))\in E(G)$ and $(T(y,x),T(v,u))\in E(G^{-1})$.

(III) $(X\times X)^{fg}_{ST}=(X\times X)_{Sf}\cap(X\times X)_{Tg}$

ST are said to be $G-fg$ contraction if

(i) f,g are edge preserving respectively. i.e., $(fx,gu)\in E(G),(fy,gv)\in E(G^{-1})$

implies $(f(fx),g(gu))\in E(G)$ and $(f(fy),g(gv))\in E(G^{-1})$.

(ii) S,T are $fg-$edge preserving. i.e., $(fx,gu)\in E(G),(fy,gv)\in E(G^{-1})$

implies $(S(x,y),T(u,v))\in E(G)$ and $(S(y,x),T(v,u))\in E(G^{-1})$

(iii) for all $x, y, u, v \in X$ and for $(fx, gu), (S(x,y), T(u,v)) \in E(G)$

And $(fy, gv), (S(y,x), T(v,u)) \in E(G^{-1})$

$$d(S(x,y),T(u,v)) \leq \max\{\kappa(\psi(fx,gu)), \kappa(\psi(S(x,y),T(u,v)))\} [\psi(fx,gu) - \psi(S(x,y),T(u,v))] - \max\{\kappa(\phi(fy,gv)), \kappa(\phi(S(y,x),T(v,u)))\} [\phi(fy,gv) - \psi(S(y,x),T(v,u))]$$

where $\psi, \phi: X \times X \to [0,\infty)$ are lower semi continuous functions.

Theorem 1.4 *Let $S,T: X \times X \to X$ and $f,g: X \to X$. Suppose that S,T are fg-edge preserving and satisfies $G - fg$ contraction.*

Let $S(X \times X) \subseteq f(X)$ and $T(X \times X) \subseteq g(X)$. Also let $\{x_{2n}\}$,$\{y_{2n}\}$,$\{u_{2n}\}$ and $\{v_{2n}\}$ be sequences in the metric space (X,d) endowed with a directed graph G. Then the following statements are true.

(i) $(fx, gu) \in E(G)$ and $(fy, gv) \in E(G^{-1})$ implies

$(S(x_{2n}, y_{2n}), T(u_{2n+1}, v_{2n+1})) \in E(G)$

and $(S(y_{2n}, x_{2n}), T(v_{2n+1}, u_{2n+1})) \in E(G^{-1})$, $\forall n \in N$;

(ii) $(x,y) \in (X \times X)_{ST}^{fg} \Rightarrow (x_{2n+1}, y_{2n+1}) \in (X \times X)_{ST}^{fg}, \forall n \in N$;

(iii) $\{\Omega_{2n}\}$ and $\{\eta_{2n}\}$ are cauchy sequences and there exists $x^*, y^* \in X$ such that $\Omega_{2n} \to x^*$ and $\eta_{2n} \to y^*$.

Theorem 1.5 *In addition to 1.4, assume that f,g are G-continuous and (i) f commutes with S and g commutes with T $[or]$ (ii) (X,d,G) has the property (A)*

Then $CCoin(_{ST}^{fg}) \neq \phi$ iff $(X \times X)_{ST}^{fg} \neq \phi$.

Theorem 1.6 *Suppose that hypothesis of 1.5 holds. Besides, let for every $(a^*, b^*), (c^*, d^*) \in (X \times X)$, there exists $(u,v) \in (X \times X)$*

such that

$(S(a^*, b^*), T(u,v)) \in E(G)$, $(S(b^*, a^*), T(v,u)) \in E(G^{-1})$ and

$(S(c^*, d^*), T(u,v)) \in E(G)$, $(S(d^*, c^*), T(v,u)) \in E(G^{-1})$

Also

$(S(u,v), T(a^*, b^*)) \in E(G)$, $(S(v,u), T(b^*, a^*)) \in E(G^{-1})$ and

$(S(u,v), T(c^*, d^*)) \in E(G)$, $(S(v,u), T(d^*, c^*)) \in E(G^{-1})$.

Then S, T, f and g have a unique CCFP.

Now we prove our main results.

RESULTS AND DISCUSSIONS

Definition 2.1 *Let $f,g: X \to X$. The two mappings $S,T: X \times X \to X$ are said to be $G - (fg)_1$ contraction if*

(i) f,g are edge preserving respectively. i.e., $(fx, gu) \in E(G), (fy, gv) \in E(G^{-1})$

implies $(f(fx), g(gu)) \in E(G)$ and $(f(fy), g(gv)) \in E(G^{-1})$

(ii) S,T are fg −edge preserving. i.e., $(fx, gu) \in E(G), (fy, gv) \in E(G^{-1})$

implies $(S(x,y), T(u,v)) \in E(G)$ and $(S(y,x), T(v,u)) \in E(G^{-1})$

(iii) $d(S(x,y), T(u,v)) \leq \frac{k}{2}[d((fx,gu)) + d((fy,gv))]$, $k \in [0, \frac{1}{2})$ is contraction constant of ST, where $(fx, gu) \in E(G), (fy, gv) \in E(G^{-1})$ for all $x, y, u, v \in X$.

Theorem 2.1 *Let $S,T: X \times X \to X$ and $f,g: X \to X$. Suppose that S,T are fg-edge preserving and satisfies $G - (fg)_1$ contraction.*

Let $S(X \times X) \subseteq f(X)$ and $T(X \times X) \subseteq g(X)$. Also let $\{x_{2n}\}$,$\{y_{2n}\}$,$\{u_{2n}\}$ and $\{v_{2n}\}$ be sequences in the metric space (X,d) endowed with a directed graph G. Then the following statements are true.

(i) $(fx, gu) \in E(G)$ and $(fy, gv) \in E(G^{-1})$ implies

$(S(x_{2n}, y_{2n}), T(u_{2n+1}, v_{2n+1})) \in E(G)$ and $(S(y_{2n}, x_{2n}), T(v_{2n+1}, u_{2n+1})) \in E(G^{-1})$, $\forall n \in N$;

(ii) $(x,y) \in (X \times X)_{ST}^{fg} \Rightarrow (x_{2n+1}, y_{2n+1}) \in (X \times X)_{ST}^{fg}, \forall n \in N$;

(iii) $\{Z_{2n}\}$ and $\{W_{2n}\}$ are cauchy sequences and there exists $x^*, y^* \in X$ such that $\Omega_{2n} \to x^*$ and $\eta_{2n} \to y^*$

Proof: We have $S(X \times X) \subseteq g(X)$ and $T(X \times X) \subseteq f(X)$ so let us define the following sequences

$$z_{2n} = gx_{2n+1} = S(x_{2n}, y_{2n}),$$
$$w_{2n} = gy_{2n+1} = S(y_{2n}, x_{2n}),$$
$$z_{2n+1} = fx_{2n+2} = T(x_{2n+1}, y_{2n+1}),$$
$$w_{2n+1} = fy_{2n+2} = T(y_{2n+1}, x_{2n+1}), \quad n = 0,1,2,\cdots.$$

the rest of the proof followed Theorem1 conditions (i) and (ii) proof.

Theorem 2.2 *Let (X,d) be a metric space endowed with a directed graph G. Let $S,T: X \times X \to X$ are $G - fg$ −contraction with contraction constant $k \in [0, \frac{1}{2})$ and $S(X \times X) \subseteq g(X)$ and $T(X \times X) \subseteq f(X)$. Also suppose that $(x_{2n}), (y_{2n})$ be sequences in X. Then, for $(x,y) \in (X \times X)$, there exist $r(x,y) \geq 0$ such that $d(fx_{2n}, gx_{2n+1}) \leq \frac{k^{2n-1}}{2} r(x,y)$ and $d(fy_{2n}, gy_{2n+1}) \leq \frac{k^{2n-1}}{2} r(x,y)$.*

Proo: . Let $(x,y) \in (X \times X)_{Tg}$

$$\Rightarrow (gx, T(x,y)) \in E(G) \text{ and } (gy, T(y,x)) \in E(G^{-1})$$
$$\Rightarrow (gx_1, T(x_1,y_1)) \in E(G) \text{ and } (gy_1, T(y_1,x_1)) \in$$

by theorem2.1 and edge preserving property , we have

$(T(x_{2n}, y_{2n}), S(x_{2n+1}, y_{2n+1}) \in E(G)$

$$\Rightarrow (fx_{2n+1}, gx_{2n+2}) \in E(G).$$

By $G - (fg)_1$ contraction

$$d(fx_{2n+1}, gx_{2n+2}) = d(T(x_{2n}, y_{2n}), S(x_{2n+1}, y_{2n+1})) \leq \frac{k}{2}[d(fx_{2n}, gx_{2n+1}) + d(fy_{2n}, gy_{2n+1})]$$

$$\leq \frac{k}{2}[d(T(x_{2n-1}, y_{2n-1}), S(x_{2n}, y_{2n})) + d(T(y_{2n-1}, x_{2n-1}), S(y_{2n}, x_{2n}))$$

$$\leq \frac{k^2}{2}[d(fx_{2n-1}, gx_{2n}) + d(fy_{2n-1}, gy_{2n})]$$

by repeating the above process, we have

$d(fx_{2n+1}, gx_{2n+2}) \leq \frac{k^{2n}}{2} r(x, y)$,

where $r(x, y) = d(fx_0, gx_1) + d(fy_0, gy_1)$

smilarlly we can prove that

$d(fy_{2n+1}, gy_{2n+2}) \leq \frac{k^{2n}}{2} r(x, y)$, where $r(x, y)$

$= d(fy_0, gy_1) + d(fx_0, gx_1)$.

Theorem 2.3 *Let (X, d) be a complete metric space endowed with a directed graph G. Let $S, T: X \times X \to X$ are $G-fg$ −contraction with contraction constant $k \in [0, \frac{1}{2})$ and $S(X \times X) \subseteq g(X)$ and $T(X \times X) \subseteq f(X)$. Also let $(x_{2n}), (y_{2n})$ be sequences in X. Then for each $(x, y) \in (X \times X)$, there exists $x^*, y^* \in X$ such that $fx_{2n} \to x^*$ and $gy_{2n} \to y^*$ as $n \to \infty$.*

Proof: Let $(x, y) \in (X \times X)_{Tg}$.

Then using theorem(1.5)

$d(fx_{2n+1}, gx_{2n+2}) \leq \frac{k^{2n}}{2} r(x, y)$

and

$d(fy_{2n+1}, gy_{2n+2}) \leq \frac{k^{2n}}{2} r(x, y)$,

forall $n \in N$ where $k \in [0, \frac{1}{2})$.

Now for $m > n$, we have

$$d(z_{2n}, z_{2m+1})$$

$$= d(z_{2n}, z_{2n+1}) + d(z_{2n+1}, z_{2n+2}) + \cdots + d(z_{2m}, z_{2m+1})$$

$$\leq d(fx_{2n+1}, gx_{2n+2}) + d(fx_{2n+2}, gx_{2n+3}) + \cdots + d(fx_{2m+1}, gx_{2m+2})$$

$$\leq \frac{k^{2n}}{2} r(x, y) + \frac{k^{2n+1}}{2} r(x, y) + \cdots + \frac{k^{2m}}{2} r(x, y)$$

$$\leq \left(\frac{k^{2n}}{2} + \frac{k^{2n+1}}{2} + \cdots + \frac{k^{2m}}{2}\right) r(x, y)$$

$$\leq \frac{k^{2n}}{2} r(x, y)[1 + k + k^2 + \cdots + k^{2m-2n}]$$

$$= \frac{k^{2n}}{2} r(x, y)\left[1 - \frac{k^{2m-2n}}{1} - k\right].$$

as m, n are large and $k \in [0, \frac{1}{2})$

Therefore $d(z_{2n}, z_{2m+1}) \to 0$ as $m, n \to \infty$.

This shows that z_{2n} is a cauchy sequence.

Similarly we can prove that w_{2n} is a cauchy sequence.

Since (X, d) is a complete so there exists $u, v \in X$ such that $z_{2n} \to u$ and $w_{2n} \to v$.

Therefore $\lim_{n\to\infty} z_{2n} = u$ and $\lim_{n\to\infty} w_{2n} = v$.

Theorem 2.4 *Suppose (X, p) is complete endowed with a directed graph G. Let $S: X \times X \to X$ and $T: X \times X \to X$ are satisfies $G - fg$ contraction with contraction constant $k \in [0, \frac{1}{2})$ and $S(X \times X) \subseteq f(X)$, $T(X \times X) \subseteq g(X)$. Let f is G continuous and commutes with S and g is G continuous and commutes with T.Also,assume either*

(i) S, T are G continuous (ii) (X, p, G) has the property (A)

Then $CCoin(Sf) \neq \phi$ iff $(X \times X)_{Sf} \neq \phi$ and $CCoin(Tg) \neq \phi$ iff $(X \times X)_{Tg} \neq \phi$.

Proof: Suppose $CCoin(Sf) \neq \phi$,

Then there exists $(u, v) \in CCoin(Sf)$.

i.e., $fu = S(u, v)$ and $fv = S(v, u)$.

So $(fu, fu) = (fu, S(u, v)) \in E(G)$

and $(fv, fv) = (fv, S(v, u)) \in E(G^{-1})$

$$\Rightarrow (u, v) \in (X \times X)_{Sf}$$

$$\Rightarrow (X \times X)_{Sf} \neq \phi.$$

Next, Let us assume that $(X \times X)_{Sf} \neq \phi$.

Then there exists some $(x_0, y_0) \in (X \times X)_{Sf}$

so we have $(fx_0, S(x_0, y_0)) \in E(G)$ and $(fy_0, S(y_0, x_0)) \in E(G^{-1})$

Then by theorem (2.1), condition (ii), there exists a sequence $\{n_i\}_{i\in N}$ of positive integers such that $(S(x_{2ni}, y_{2ni}), T(x_{2ni+1}, y_{2ni+1})) \in E(G)$ and

$(S(y_{2ni}, x_{2ni}), T(y_{2ni+1}, x_{2ni+1})) \in E(G^{-1})$.

Then by theorem (2.2) $\lim_{n\to\infty} S(x_{2ni}, y_{2ni}) \to u$ and $\lim_{n\to\infty} T(x_{2ni+1}, y_{2ni+1}) \to v$.

Since f is G continuous so

$\lim_{n\to\infty} f(S(x_{2ni}, y_{2ni})) \to fu$

and

$\lim_{n\to\infty} f(T(x_{2ni+1}, y_{2ni+1})) \to fv$.

Since (S, f) are commute so we have $f(S(x_{2ni}, y_{2ni})) = S(fx_{2ni}, fy_{2ni})$. and $f(S(y_{2ni}, x_{2ni})) = S(fy_{2ni}, fx_{2ni})$

Now

$$\lim_{n\to\infty} f(S(x_{2ni}, y_{2ni})) = S \lim_{n\to\infty} (fx_{2ni}, fy_{2ni})$$

$$\Rightarrow fu = S(u, v)$$

Similarlly ,

$$\lim_{n\to\infty} f(S(y_{2ni}, x_{2ni})) = S \lim_{n\to\infty} (fy_{2ni}, fx_{2ni})$$

$$\Rightarrow fv = S(v, u)$$

In the same way we can prove that $gu = T(u, v)$ and $gv = T(v, u)$.

This shows that (u, v) is the coupled coincidence point S, T, f and g.

Next assume that (X, p, G) satisfies property (A).

Since $T(x_{2ni+1}, y_{2ni+1}) \to u$ as $i \to \infty$ and $T(y_{2ni+1}, x_{2ni+1}) \to v$ as $i \to \infty$

also $(S(x_{2ni}, y_{2ni}), T(x_{2ni+1}, y_{2ni+1})) \in E(G)$

and $(S(y_{2ni}, x_{2ni}), T(y_{2ni+1}, x_{2ni+1})) \in E(G^{-1})$

so by property (A) $(S(x_{2ni}, y_{2ni}), u) \in E(G)$ and $(S(y_{2ni}, x_{2ni}), v) \in E(G^{-1})$.

Now

$d(fu, S(u,v))$

$= d(fu, f(S(x_{2ni}, y_{2ni}))) + d(f(S(x_{2ni}, y_{2ni}), S(u,v)) = d\left(f\left(T(x_{2ni+1}, y_{2ni+1})\right), f\left(S(x_{2ni}, y_{2ni})\right)\right)$

$+d(f(S(x_{2ni}, y_{2ni}), S(u,v))$

$\leq d\left(T(x_{2ni+1}, y_{2ni+1}), S(x_{2ni}, y_{2ni})\right) + d\left(S(fx_{2ni}, fy_{2ni}), S(u,v)\right)$

$\leq \frac{k}{2}[d(fx_{2ni+1}, gx_{2ni+1}) + d(fy_{2ni+1}, gy_{2ni+1})]$

$+d(S(fx_{2ni}, fy_{2ni}), S(u,v))$

Since $k \in [0, \frac{1}{2})$, As $i \to \infty$,

$$d(fu, S(u,v)) = \frac{k}{2}[d(u,u) + d(v,v)] + d(S(u,v), S(u,v)) = 0.$$

Therefore $fu = S(u,v)$. Similarly we can prove that $fv = S(v,u)$.

In this way we can prove that $gu = T(u,v)$ and $gv = T(v,u)$.

Finally to uniqueness of the common coupled fixed point can be proved using theorem 3 via $G - (fg)_1$ contraction.

Application to Integral equations

To discuss the application of our main results we establish an existence theorem in a metric space with graph for the solution of the integral equations.

Consider the following integral equations:

$$x(t) = \int_0^T f(t, x(s), y(s))ds, \quad t \in [0,T]$$

$$y(t) = \int_0^T f(t, y(s), x(s))ds, \quad t \in [0,T]$$

Where T is a positive real number and $f: [0,T] \times \Re \times \Re \to \Re$.

Consider $X = C([0,T], \Re)$. Define $d: X \times X \to \Re$ as $d(x,y) = \max\{x(t), y(t)\}$.

Clearly d is a metric on X.

Define a graph G using the following partial relation.

$x \leq y \Leftrightarrow x(t) \leq y(t)$, for all $x, y \in X$ and for any $t \in [0,T]$.

So, we have

$E(G) = \{(x,y) \in X \times X: x \leq y$

and $E(G^{-1}) = \{(x,y) \in X \times X: y \leq x$

Also $\Delta(X \times X) \subseteq E(G)$ and (X, d, G) has property (A).
Clearly (X, d) is a complete metric space with a directed graph G.

Theorem 3.1 Suppose for the integral equation,

(i) $f: [0,T] \times \Re \times \Re \to \Re$ is continuous;
(ii) for all $t \in [0,T]$ and $x, y, u, v \in \Re$ with $x \leq u, v \leq y$, $f(t,x,y) \leq f(t,u,v)$;
(iii) for each $t \in [0,T]$ and $x, y, u, v \in \Re$ with $x \leq u, v \leq y$, there exists $k \in [0,1)$ such that $\max\{f(t,x,y), f(t,u,v)\} \leq \frac{k}{T}\max\{x(t), u(t), y(t), v(t)\}$.
(iv) there exists $(x_0, y_0) \in X \times X$ such that for all $t \in [0,T]$,

$$x(t) \leq \int_0^T f(t, x_0(s), y_0(s))ds, \quad t \in [0,T]$$

$$\int_0^T f(t, y_0(s), x_0(s))ds \leq y(t), \quad t \in [0,T]$$

Then there exists at least one solution of the given integral equation.

CONCLUSIONS:

By defining the new $G - (fg)_1$ contraction we obtained a unique common coupled fixed point for mapping and obtained solution of integral equation.

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